## Chapter 7

## Conservation of Energy and Momentum

## Practice Problem Solutions

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## 1. Conceptualize the Problem

- The roller coaster car starts at a position of high gravitational potential with an initial speed, so it has both gravitational potential energy and kinetic energy.
- Define the system as the roller coaster car and the roller coaster track.
- Assume that the system is isolated.
- The law of conservation of energy can be applied.
- Let the subscript " 1 " indicate the roller coaster car's initial position.


## Identify the Goal

The speed of the roller coaster car at point $\mathrm{A}, v_{\mathrm{A}}$
Identify the Variables
Known Implied

$$
\begin{aligned}
& v_{1}=4.0 \frac{\mathrm{~m}}{\mathrm{~s}} \\
& h_{1}=12.0 \mathrm{~m} \\
& h_{\mathrm{A}}=4.0 \mathrm{~m}
\end{aligned}
$$

## Develop a Strategy

State the law of conservation of energy. Expand by replacing $E$ with the expression that defines that type of energy. Solve for the speed.
Substitute known values and solve.

## Calculations

$$
\begin{aligned}
& E_{\mathrm{k}}^{\prime}+E_{\mathrm{p}}^{\prime}=E_{\mathrm{k}}+E_{\mathrm{p}} \\
& \frac{1}{2} m v_{\mathrm{A}}^{2}+m g h_{\mathrm{A}}=\frac{1}{2} m v_{1}^{2}+m g h_{1} \\
& v_{\mathrm{A}}^{2}=v_{1}^{2}+2 g\left(h_{1}-h_{\mathrm{A}}\right) \\
& v_{\mathrm{A}}=\sqrt{v_{1}^{2}+2 g\left(h_{1}-h_{\mathrm{A}}\right)} \\
& v_{\mathrm{A}}=\sqrt{\left(4.0 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}\right)+2\left(9.81 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}\right)(12.0 \mathrm{~m}-4.0 \mathrm{~m})} \\
& v_{\mathrm{A}}= \pm 13.1514 \frac{\mathrm{~m}}{\mathrm{~s}} \\
& v_{\mathrm{A}} \cong 13 \frac{\mathrm{~m}}{\mathrm{~s}}
\end{aligned}
$$

The initial speed was positive,

## Unknown

$v_{\mathrm{A}}$
so choose the positive root. The
speed of the roller coaster car
at position A was about $13 \frac{\mathrm{~m}}{\mathrm{~s}}$.
Validate the Solution
Check the units: $\left(\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^{2}+\frac{\mathrm{m}}{\mathrm{s}^{2}} \cdot \mathrm{~m}\right)^{\frac{1}{2}}=\left(\frac{\mathrm{m}^{2}}{\mathrm{~s}^{2}}\right)^{\frac{1}{2}}=\frac{\mathrm{m}}{\mathrm{s}}$.
The roller coaster car's speed increases to about $13 \frac{\mathrm{~m}}{\mathrm{~s}}$, from $4.0 \frac{\mathrm{~m}}{\mathrm{~s}}$, after a height change of 8.0 m , which seems reasonable. Notice that the answer is independent of the mass of the roller coaster.

## 2. Conceptualize the Problem

- The roller coaster car starts at a position of high gravitational potential with an initial speed, so it has both gravitational potential energy and kinetic energy.
- Define the system as the roller coaster car and the roller coaster track.
- Assume that the system is isolated.
- The law of conservation of energy can be applied.
- Let the subscript " 1 " indicate the roller coaster car's initial position.
- Note that calculations for the conservation of energy can be made with information from either point A (from the previous question) or point 1 ; point 1 is chosen for convenience.


## Identify the Goal

The height of the roller coaster car at point $\mathrm{B}, h_{\mathrm{B}}$

## Identify the Variables

| Known | Implied | Unknown |
| :--- | :--- | :---: |
| $v_{\mathrm{B}}=10.0 \frac{\mathrm{~m}}{\mathrm{~s}}$ |  | $h_{\mathrm{B}}$ |
| $v_{1}=4.0 \frac{\mathrm{~m}}{\mathrm{~s}}$ |  |  |
| $h_{1}=12.0 \mathrm{~m}$ |  |  |

## Develop a Strategy

State the law of conservation of energy. Expand by replacing $E$ with the expression that defines that type of energy. Solve for the height.
Notice that the mass cancels out and the answer is independent of this variable.
Substitute known values and solve.
The height of the roller coaster track
at position B is about 7.7 m .

## Calculations

$$
\begin{aligned}
E_{\mathrm{k}}^{\prime}+E_{\mathrm{p}}^{\prime} & =E_{\mathrm{k}}+E_{\mathrm{p}} \\
\frac{1}{2} m v_{\mathrm{B}}^{2}+m g h_{\mathrm{B}} & =\frac{1}{2} m v_{1}^{2}+m g h_{1} \\
h_{\mathrm{B}} & =\frac{\frac{1}{2} v_{1}^{2}+g h_{1}-\frac{1}{2} v_{\mathrm{B}}^{2}}{g}
\end{aligned}
$$

## Validate the Solution

Check the units: $\frac{\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^{2}+\frac{\mathrm{m}}{\mathrm{s}^{2}} \cdot \mathrm{~m}-\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^{2}}{\frac{\mathrm{~m}}{\mathrm{~s}^{2}}}=\mathrm{m}$.
It is expected that $B$ will be higher than $A$, and it is.

## 3. Conceptualize the Problem

- The roller coaster car starts at a position of high gravitational potential with an initial speed, so it has both gravitational potential energy and kinetic energy.
- Define the system as the roller coaster car and the roller coaster track.
- Assume that the system is isolated.
- The law of conservation of energy can be applied.
- Let the subscript " 1 " indicate the roller coaster car's initial position.
- Note that calculations for the conservation of energy can be made with information from either point $A$ (from question 10) or point 1 ; point 1 is chosen for convenience.


## Identify the Goal

The height of the roller coaster car at point $\mathrm{B}, h_{\mathrm{B}}$

## Identify the Variables

Known Implied Unknown

$$
\begin{aligned}
v_{\mathrm{B}} & =12.5 \frac{\mathrm{~m}}{\mathrm{~s}} \\
v_{1} & =4.0 \frac{\mathrm{~m}}{\mathrm{~s}} \\
h_{1} & =12.0 \mathrm{~m}
\end{aligned}
$$

## Develop a Strategy

State the law of conservation of energy. Expand by replacing $E$ with the expression that defines that type of energy. Solve for the height.
Notice that the mass cancels out and the answer is independent of this variable.
Substitute known values and solve. $\quad h_{B}=\frac{\frac{1}{2}\left(4.0 \frac{\mathrm{~m}}{\mathrm{~s}}\right)^{2}+\left(9.81 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}\right)(12.0 \mathrm{~m})-\frac{1}{2}\left(12.5 \frac{\mathrm{~m}}{\mathrm{~s}}\right)^{2}}{9.81 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}}$
The height of the roller coaster track at position B is about 4.8 m .

## Calculations

$$
\begin{aligned}
E_{\mathrm{k}}^{\prime}+E_{\mathrm{p}}^{\prime} & =E_{\mathrm{k}}+E_{\mathrm{p}} \\
\frac{1}{2} m v_{\mathrm{B}}^{2}+m g h_{\mathrm{B}} & =\frac{1}{2} m v_{1}^{2}+m g h_{1} \\
h_{\mathrm{B}} & =\frac{\frac{1}{2} v_{1}^{2}+g h_{1}-\frac{1}{2} v_{\mathrm{B}}^{2}}{g}
\end{aligned}
$$

Validate the Solution
Check the units: $\frac{\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^{2}+\frac{\mathrm{m}}{\mathrm{s}^{2}} \cdot \mathrm{~m}-\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^{2}}{\frac{\mathrm{~m}}{\mathrm{~s}^{2}}}=\mathrm{m}$.
The roller coaster car travels faster at B in this problem than in the previous problem, so it is expected that the height of $B$ will be lower in this problem (though still higher than A), and it is.

## 4. Frame the Problem

- Since there is no friction acting the conservation of mechanical energy applies.
- From the conservation of mechanical energy, the kinetic energy of the ball when thrown (at the bottom) will be completely transformed into gravitational potential energy when it reaches the top.
- The mass is not needed since there is no friction acting, and thus mechanical energy is conserved. Mathematically, the mass cancels out on each side of the equation.


## Identify the Goal

The maximum height reached by the ball.
Variables and Constants

| Known | Implied |
| :--- | :--- |
| $v_{1}=10.0 \mathrm{~m} / \mathrm{s}$ | $g=9.81 \frac{\mathrm{~N}}{\mathrm{~kg}}$ |

## Strategy

Use the formula for conservation of mechanical energy.
Elaborate.
Cancel the mass on both sides.

Rearrange to solve for $\Delta h$

Substitute in the variables.

Simplify.

## Unknown

$\Delta h$

## Calculations

$E_{\mathrm{g}_{\text {TOP }}}=E_{\mathrm{k}_{\text {BOTTOM }}}$
$m g \Delta h=\frac{1}{2} m v^{2}$
$g \Delta h=\frac{1}{2} v^{2}$
$\Delta h=\frac{\frac{1}{2} v^{2}}{g}$
$\Delta h=\frac{\frac{1}{2}\left(10.0 \frac{\mathrm{~m}}{\mathrm{~s}}\right)^{2}}{9.81 \frac{\mathrm{~N}}{\mathrm{~kg}}}$
$\Delta h=5.10 \frac{\frac{\mathrm{~m}^{2}}{\mathrm{~s}^{2}}}{\frac{\mathrm{~N}}{\mathrm{~kg}}}$

An $\frac{\frac{m^{2}}{s^{2}}}{\frac{N}{k g}}$ is equivalent to an $\mathrm{m} . \quad \Delta h=5.10 \mathrm{~m}$
The maximum height reached by the ball is 5.10 m .
Validate
The ball's kinetic energy is transformed into gravitational potential energy as it rises. The unit for height is the m .

## 5. Frame the Problem

- When the wrecking ball is inclined, the vertical height relative to its lowest position determines its gravitational potential energy.
- The length of the cable and the angle of incline determine the vertical height.
- The gravitational potential energy at the highest position will be transformed into kinetic energy at its lowest position.


## Identify the Goal

The gravitational potential energy of the wrecking ball at its highest position, its kinetic energy and speed at the lowest position.

Variables and Constants

| Known | Implied | Unknown |
| :--- | :--- | :--- |
| $m=315 \mathrm{~kg}$ | $g=9.81 \frac{\mathrm{~N}}{\mathrm{~kg}}$ | $\Delta h$ |
| $1=10.0 \mathrm{~m}$ |  | $E_{\mathrm{g}}$ |
| $\Theta=30.0^{\circ}$ |  | $E_{\mathrm{k}}$ |
|  |  | $v$ |

PART I
Strategy
Use the cosine function to find the vertical component of the length.
Substitute in the variables.
Simplify.
Subtract to find the vertical height of the wrecking ball.
Use the formula for gravitational potential energy.
Substitute in the variables.
Simplify.
An $\mathrm{N} \cdot \mathrm{m}$ is equivalent to a J .

## Calculations

Adjacent $=$ Hypotenuse $\cdot \cos \Theta$

Adjacent $=(10.0 \mathrm{~m})\left(\cos 30.0^{\circ}\right)$
Adjacent $=8.66 \mathrm{~m}$
$\Delta h=10.0 \mathrm{~m}-8.66 \mathrm{~m}=1.34 \mathrm{~m}$
$E_{\mathrm{g}}=m g \Delta h$
$E_{\mathrm{g}}=(315 \mathrm{~kg})\left(9.81 \frac{\mathrm{~N}}{\mathrm{~kg}}\right)(1.34 \mathrm{~m})$
$E_{\mathrm{g}}=4140.8 \mathrm{~N} \cdot \mathrm{~m}$
$E_{\mathrm{g}}=4140.8 \mathrm{~J}$

PART II
Strategy
Use the conservation of mechanical energy.
Substitute in the variables.

## PART III

Strategy
Use the formula for kinetic energy
to find the speed.
Substitute in the variables.

## Calculations

$$
\begin{aligned}
& E_{\mathrm{g}_{\text {TOP }}}=E_{\mathrm{k}_{\text {BOTTOM }}} \\
& E_{\mathrm{k}_{\text {BOTTOM }}}=4140.8 \mathrm{~J}
\end{aligned}
$$

## Calculations

$v=\sqrt{\frac{2 E_{\mathrm{k}}}{\mathrm{m}}}$
$v=\sqrt{\frac{2(4140.8 \mathrm{~J})}{315 \mathrm{~kg}}}$

Simplify.
$v=5.13\left(\sqrt{\frac{\mathrm{~J}}{\mathrm{~kg}}}\right)$
A $\sqrt{\frac{\mathrm{J}}{\mathrm{kg}}}$ is equivalent to an $\frac{\mathrm{m}}{\mathrm{s}}$.
$v=5.13 \frac{\mathrm{~m}}{\mathrm{~s}}$

## Validate

The unit for gravitational potential energy and kinetic energy is the J. The unit for speed is the $\mathrm{m} / \mathrm{s}$.

## 6. Frame the Problem

- The objects have different masses, so they will have different amounts of gravitational potential energy, even when dropped from the same height.
- The formula for gravitational potential energy applies.
- The gravitational potential energy will be transformed into kinetic energy as the objects fall because friction is negligible.


## Identify the Goal

The kinetic energy and velocity of each object just before they hit the ground.

## FOR THE LEAD BALL

## Variables and Constants

| Known | Implied | Unknown |
| :--- | :--- | :--- |
| $m=2.5 \mathrm{~kg}$ | $g=9.81 \frac{\mathrm{~N}}{\mathrm{~kg}}$ | $E_{\mathrm{k}}$ |
| $\Delta h=25 \mathrm{~m}$ |  | $v$ |

## Strategy

Use the formula for gravitational potential energy.
Substitute in the variables.
Simplify.
An $\mathrm{N} \cdot \mathrm{m}$ is equivalent to a J .
Use the conservation of mechanical energy to find the kinetic energy.
Substitute in the variables.
Use the formula for kinetic energy to find the speed.
Substitute in the variables.
Simplify.
A $\sqrt{\frac{\mathrm{J}}{\mathrm{kg}}}$ is equivalent to an $\frac{\mathrm{m}}{\mathrm{s}}$.
The lead ball has a kinetic energy of 613.1 J and a speed of $22 \mathrm{~m} / \mathrm{s}$ just before it strikes the ground.

FOR THE LEAD SHOT
Variables and Constants

| Known | Implied | Unknown |
| :--- | :--- | :--- |
| $m=0.055 \mathrm{~kg}$ | $g=9.81 \frac{\mathrm{~N}}{\mathrm{~kg}}$ | $E_{\mathrm{k}}$ |
| $\Delta h=25 \mathrm{~m}$ |  | $v$ |

## Strategy

Use the formula for gravitational potential energy.
Substitute in the variables.
Simplify.
An $\mathrm{N} \cdot \mathrm{m}$ is equivalent to a J .
Use the conservation of mechanical energy to find the kinetic energy.
Substitute in the variables.
Use the formula for kinetic energy to find the speed.
Substitute in the variables.
Simplify.
A $\sqrt{\frac{\mathrm{J}}{\mathrm{kg}}}$ is equivalent to an $\frac{\mathrm{m}}{\mathrm{s}}$.

Calculations
$E_{\mathrm{g}}=m g \Delta h$
$E_{\mathrm{g}}=(0.055 \mathrm{~kg})\left(9.81 \frac{\mathrm{~N}}{\mathrm{~kg}}\right)(25 \mathrm{~m})$
$E_{\mathrm{g}}=13.49 \mathrm{~N} \cdot \mathrm{~m}$
$E_{\mathrm{g}}=13.49 \mathrm{~J}$
$E_{\mathrm{k}_{\text {BOTTOM }}}=E_{\mathrm{g}_{\text {TOP }}}$
$E_{\mathrm{k}_{\text {воттом }}}=13.49 \mathrm{~J}$
$v=\sqrt{\frac{2 E_{\mathrm{k}}}{\mathrm{m}}}$
$v=\sqrt{\frac{2(13.49 \mathrm{Jj}}{0.055 \mathrm{~kg}}}$
$v=22\left(\sqrt{\frac{\mathrm{~J}}{\mathrm{~kg}}}\right)$
$v=22 \frac{\mathrm{~m}}{\mathrm{~s}}$

The lead shot has a kinetic energy of 13.49 J and a speed of $22 \mathrm{~m} / \mathrm{s}$ just before it strikes the ground.

## Validate

The lead ball has a greater mass and thus greater kinetic energy as compared to the lead shot. The unit for kinetic energy is the J and the unit for speed is the $\mathrm{m} / \mathrm{s}$.

## 7. Frame the Problem

- While at the top of the ramp the crate has both kinetic and gravitational potential energy.
- While at the bottom of the ramp the crate's energy is all in the form of kinetic energy.
- Mechanical energy is conserved since the ramp is frictionless.


## Identify the Goal

The length of the ramp.
Variables and Constants

| Known | Implied | Unknown |
| :--- | :--- | :--- |
| $m=32 \mathrm{~kg}$ | $g=9.81 \frac{\mathrm{~N}}{\mathrm{~kg}}$ | $E_{\mathrm{g}_{\text {TOP }}}$ |
| $v_{\text {TOP }}=3.2 \mathrm{~m} / \mathrm{s}$ | $E_{\mathrm{g}_{\text {BOTTOM }}}=0 \mathrm{~J}$ | $E_{\mathrm{k}_{\text {TOP }}}$ |
| $v_{\text {BOTTOM }}=9.7 \mathrm{~m} / \mathrm{s}$ |  | $E_{\mathrm{k}_{\text {BOTTOM }}}$ |
| $\Theta=25^{\circ}$ |  | $E_{\mathrm{T}}$ |

## Strategy

Find the kinetic energy at the bottom.
Substitute in the variables.
Simplify.

## Calculations

$E_{\mathrm{k}}=\frac{1}{2} m v^{2}$
$E_{\mathrm{k}}=\frac{1}{2}(32 \mathrm{~kg})\left(9.7 \frac{\mathrm{~m}}{\mathrm{~s}}\right)^{2}$
A $\mathrm{kg} \cdot \frac{\mathrm{m}^{2}}{\mathrm{~s}^{2}}$ is equivalent to a J .
Find the mechanical energy at the bottom.
Substitute in the variables.
$E_{\mathrm{k}}=1505.44 \mathrm{~kg} \cdot \frac{\mathrm{~m}^{2}}{\mathrm{~s}^{2}}$
$E_{\mathrm{k}}=1505.44 \mathrm{~J}$
$E_{\mathrm{T}}=E_{\mathrm{k}}+E_{\mathrm{g}}$

Simplify.
$E_{\mathrm{T}}=1505.44 \mathrm{~J}+0 \mathrm{~J}$
$E_{\mathrm{T}}=1505.44 \mathrm{~J}$

Find the kinetic energy at the top. $\quad E_{\mathrm{k}}=\frac{1}{2} m v^{2}$
Substitute in the variables.
$E_{\mathrm{k}}=\frac{1}{2}(32 \mathrm{~kg})\left(3.2 \frac{\mathrm{~m}}{\mathrm{~s}}\right)^{2}$
Simplify.
$E_{\mathrm{k}}=163.84 \mathrm{~kg} \cdot \frac{\mathrm{~m}^{2}}{\mathrm{~s}^{2}}$
A $\mathrm{kg} \cdot \frac{\mathrm{m}^{2}}{\mathrm{~s}^{2}}$ is equivalent to a J .
$E_{\mathrm{k}}=163.84 \mathrm{~J}$
Use conservation of mechanical energy

$$
E_{\mathrm{g}}=E_{\mathrm{T}}-E_{\mathrm{k}}
$$

to find gravitational potential energy at the top.
Substitute in the variables.
$E_{\mathrm{g}}=1505.44 \mathrm{~J}-163.84 \mathrm{~J}$
Simplify.
Use the formula for gravitational potential energy to find the height.
Substitute in the variables.
Simplify.
A $\frac{\mathrm{J}}{\mathrm{N}}$ is equivalent to an m .
Use the definition of the sine function
to find the length of the ramp.
Substitute in the variables.
Simplify.
$E_{\mathrm{g}}=1341.6 \mathrm{~J}$
$\Delta h=\frac{E_{g}}{m g}$
$\Delta h=\frac{1341.6 \mathrm{~J}}{(32 \mathrm{~kg})\left(9.81 \frac{\mathrm{~N}}{\mathrm{~kg}}\right)}$
$\Delta h=4.27 \frac{\mathrm{~J}}{\mathrm{~N}}$
$\Delta h=4.27 \mathrm{~m}$
$l=\frac{\Delta b}{\sin \Theta}$

The ramp is $1.0 \times 10^{1} \mathrm{~m}$ long.
Validate
The crate had both kinetic and gravitational potential energy at the top of the ramp, and only kinetic energy at the bottom of the ramp. The unit for length is the m .

## 8. Frame the Problem

- The wrench has both kinetic energy and gravitational potential energy while at the level of the eighth floor.
- The mechanical energy of the wrench is conserved.


## PART I

## Identify the Goal

The number of floors in the building.

| Variables and Constants |  |  |
| :--- | :--- | :--- |
| Known | Implied |  |
| $m=0.125 \mathrm{~kg}$ | $g=9.81 \frac{\mathrm{~N}}{\mathrm{~kg}}$ | $E_{\mathrm{k}}$ |
| $v=33.1 \mathrm{~m} / \mathrm{s}$ |  | $\Delta h$ |
|  |  | $E_{\mathrm{g}}$ |

## Strategy

## Calculations

Find the kinetic energy while at the eighth floor using the kinetic energy formula.
Substitute in the variables.

$$
\text { A } \mathrm{kg} \cdot \frac{\mathrm{~m}^{2}}{\mathrm{~s}^{2}} \text { is equivalent to a } \mathrm{J} \text {. }
$$

$$
\begin{aligned}
E_{\mathrm{k}}= & \frac{1}{2} m v^{2} \\
E_{\mathrm{k}}= & \frac{1}{2}(0.125 \mathrm{~kg})\left(33.1 \frac{\mathrm{~m}}{\mathrm{~s}}\right)^{2} \\
E_{\mathrm{k}}= & 68.48 \mathrm{~kg} \cdot \frac{\mathrm{~m}^{2}}{\mathrm{~s}^{2}} \\
E_{\mathrm{k}}= & 68.48 \mathrm{~J} \\
\Delta h= & \text { height of first floor } \\
& +7 \text { other floors }
\end{aligned}
$$

Find the height to the eighth floor.

Substitute in the variables
Simplify.
Find the gravitational potential energy of the wrench while at the eighth floor.
Substitute in the variables.
Simplify.
An $\mathrm{N} \cdot \mathrm{m}$ is equivalent to a J .
Use the formula for total mechanical energy.
Substitute in the variables.
Simplify.
The total mechanical energy is conserved, therefore it is the same value while the wrench is at the top floor.
The kinetic energy while at the top floor is zero because it is not moving.
Use the formula for total mechanical energy to find the gravitational potential energy while the wrench is at the top floor.
Substitute in the variables.
Simplify.
Use the formula for gravitational potential energy to find the height.
Substitute in the variables.
Simplify.
A $\frac{\mathrm{J}}{\mathrm{N}}$ is equivalent to an m .
Find the number of floors.
Substitute in the variables.
Simplify.
$\Delta h=12.0 \mathrm{~m}+(7 \times 8.00 \mathrm{~m})$
$\Delta h=68.0 \mathrm{~m}$
$E_{\mathrm{g}}=m g \Delta h$
$E_{\mathrm{g}}=(0.125 \mathrm{~kg})\left(9.81 \frac{\mathrm{~N}}{\mathrm{~kg}}\right)(68.0 \mathrm{~m})$
$E_{\mathrm{g}}=83.385 \mathrm{~N} \cdot \mathrm{~m}$
$E_{\mathrm{g}}=83.385 \mathrm{~J}$
$E_{\mathrm{T}}=E_{\mathrm{g}}+E_{\mathrm{k}}$
$E_{\mathrm{T}}=83.385 \mathrm{~J}+68.48 \mathrm{~J}$
$E_{\mathrm{T}}=147.87 \mathrm{~J}$
$E_{\mathrm{T}}=147.87 \mathrm{~J}$
$E_{\mathrm{k}}=0 \mathrm{~J}$
$E_{\mathrm{g}}=E_{\mathrm{T}}-E_{\mathrm{k}}$
$E_{\mathrm{g}}=147.87 \mathrm{~J}-0 \mathrm{~J}$
$E_{\mathrm{g}}=147.87 \mathrm{~J}$
$\Delta h=\frac{E_{\mathrm{g}}}{\mathrm{mg}}$
$\Delta h=\frac{147.87 \mathrm{~J}}{\left(0.125 \mathrm{~kg}\left(9.81 \frac{\mathrm{~N}}{\mathrm{~kg}}\right)\right.}$
$\Delta h=120.59 \frac{\mathrm{~J}}{\mathrm{~N}}$
$\Delta h=120.59 \mathrm{~m}$
\# floors $=1+\frac{(\Delta h-\text { height of firss floor })}{\text { height of other floors }}$
\# floors $=1+\frac{(120.59 \mathrm{~m}-12.0 \mathrm{~m})}{8.00 \mathrm{~m}}$
\# floors $=15$

The building has 15 floors.

## PARTS II AND III [NOTE: PART III MUST BE SOLVED BEFORE PART II] Identify the Goal

The kinetic energy and speed of the wrench just before it hit the ground.
Variables and Constants
Known Implied
$m=0.125 \mathrm{~kg} \quad g=9.81 \frac{\mathrm{~N}}{\mathrm{~kg}}$
$E_{\mathrm{T}}=147.87 \mathrm{~J}$

## Strategy

The gravitational potential energy of
Unknown
$v$
$E_{\mathrm{k}}$
the wrench is zero at the ground level.
Use the formula for total mechanical
energy to find kinetic energy.
Substitute in the variables.
Simplify.

> Calculations
> $E_{\mathrm{g}}=0 \mathrm{~J}$
> $E_{\mathrm{k}}=E_{\mathrm{T}}-E_{\mathrm{g}}$
> $E_{\mathrm{k}}=147.87 \mathrm{~J}-0 \mathrm{~J}$
> $E_{\mathrm{k}}=147.87 \mathrm{~J}$

Use the formula for kinetic energy to find the speed.
Substitute in the variables.
$v=\sqrt{\frac{2 E_{k}}{\mathrm{~m}}}$
$v=\sqrt{\frac{2(147.87 \mathrm{~J})}{0.125 \mathrm{~kg}}}$
Simplify.
$v=48.57\left(\sqrt{\frac{\mathrm{~J}}{\mathrm{~kg}}}\right)$
A $\sqrt{\frac{\mathrm{J}}{\mathrm{kg}}}$ is equivalent to an $\frac{\mathrm{m}}{\mathrm{s}}$.
$v=48.57 \frac{\mathrm{~m}}{\mathrm{~s}}$
Validate
The wrench gained speed as it fell to the ground. The unit for gravitational potential energy, kinetic energy, and total mechanical energy is the J. The unit for speed is the $\mathrm{m} / \mathrm{s}$.

## Practice Problem Solutions

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## 9. Conceptualize the Problem

- The cart is moving so it has kinetic energy.
- The spring does negative work on the cart, lowering its kinetic energy, bringing it to a complete stop.
- The cart does work on the spring, giving it elastic potential energy.
- The height of the cart does not change, so there is no change in gravitational potential energy.
- Assume that friction can be ignored.
- The law of conservation of energy applies to this problem.


## Identify the Goal

(a) The maximum distance, $x$, that the spring is compressed
(b) The speed, $v$, of the cart at the moment the spring is compressed by 0.10 m
(c) The acceleration, $a$, of the cart at the moment that the spring is compressed by 0.10 m

## Identify the Variables

| Known | Implied | Unknown |
| :--- | :---: | :--- |
| $m=1.2 \mathrm{~kg}$ | $v^{\prime}=0 \frac{\mathrm{~m}}{\mathrm{~s}}$ | $x$ |
| $v=3.6 \frac{\mathrm{~m}}{\mathrm{~s}}$ |  | $v(x=0.10 \mathrm{~m})$ |
| $k=2.00 \times 10^{2} \frac{\mathrm{~N}}{\mathrm{~m}}$ |  | $a(x=0.10 \mathrm{~m})$ |

## Develop a Strategy

Write the law of conservation of energy, including the energy quantities associated with the interaction.

$$
\begin{aligned}
& \text { Calculations } \\
& E_{\mathrm{k}}^{\prime}+E_{\mathrm{e}}^{\prime}=E_{\mathrm{k}}+E_{\mathrm{e}} \\
& E_{\mathrm{e}}=0 \mathrm{~J} \\
& E_{\mathrm{k}}^{\prime}=0 \mathrm{~J} \\
& 0 \mathrm{~J}+\mathrm{E}_{\mathrm{e}}^{\prime}=\mathrm{E}_{\mathrm{k}}+0 \mathrm{~J} \\
& E_{\mathrm{e}}^{\prime}=E_{\mathrm{k}}
\end{aligned}
$$

Initially, the spring is not compressed, so the initial elastic potential energy is zero. After the interaction, the cart stopped, so the kinetic energy is zero.

Substitute the values for energy listed by substituting the expressions for the energies.
Solve for the distance the spring is compressed.
Substitute numerical values and solve.

$$
x=\sqrt{\frac{(1.2 \mathrm{~kg})\left(3.6 \frac{\mathrm{~m}}{\mathrm{~s}}\right)^{2}}{2.00 \times 10^{2} \frac{\mathrm{~N}}{\mathrm{~m}}}}
$$

(a) The cart compresses the spring

$$
x=0.2788 \mathrm{~m}
$$ about 0.28 m .

The energy equation holds throughout

$$
\begin{aligned}
\frac{1}{2} k x^{2} & =\frac{1}{2} m v^{2} \\
x & =\sqrt{\frac{m v^{2}}{k}}
\end{aligned}
$$

$$
x \cong 0.28 \mathrm{~m}
$$

$$
\frac{1}{2} k x^{2}=\frac{1}{2} m v^{2}
$$

the whole compression due to lack of
friction, therefore the equation holds for $x=0.1 \mathrm{~m}$.
$v=\sqrt{\frac{k x^{2}}{m}}$

Solve the above equation for the speed instead of the distance.
(b) The speed of the cart at the moment
$v=\sqrt{\frac{\left(2.00 \times 10^{2} \frac{\mathrm{~N}}{\mathrm{~m}}\right)(0.10 \mathrm{~m})^{2}}{(1.2 \mathrm{~kg})}}$
$v=1.29 \frac{\mathrm{~m}}{\mathrm{~s}}$
$v \cong 1.3 \frac{\mathrm{~m}}{\mathrm{~s}}$ the spring is compressed by 0.10 m is about $1.3 \frac{\mathrm{~m}}{\mathrm{~s}}$.
Apply Hooke's law and Newton's second law.
(c) The acceleration of the cart at the moment the spring is compressed by 0.10 m is about $17 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}$.

$$
\begin{aligned}
F_{\mathrm{a}} & =k x=m a \\
a & =\frac{k x}{m} \\
a & =\frac{\left(2.00 \times 10^{2} \frac{\mathrm{~N}}{\mathrm{~m}}\right)(0.10 \mathrm{~m})}{1.2 \mathrm{~kg}} \\
a & =16.67 \frac{\mathrm{~N}}{\mathrm{~kg}} \\
a & \cong 17 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}
\end{aligned}
$$

## Validate the Solution

Check the units for (a):
$\left(\frac{\mathrm{kg}\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^{2}}{\frac{\mathrm{~N}}{\mathrm{~m}}}\right)^{\frac{1}{2}}=\left(\frac{\mathrm{kg}\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^{2} \mathrm{~m}}{\mathrm{~N}}\right)^{\frac{1}{2}}=\left(\frac{\mathrm{kg}\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^{2} \mathrm{~m}}{\mathrm{~kg} \cdot \frac{\mathrm{~m}}{\mathrm{~s}^{2}}}\right)^{\frac{1}{2}}=\left(\mathrm{m}^{2}\right)^{\frac{1}{2}}=\mathrm{m}$
The result is less than 1 m , as expected for the types of springs found in a laboratory.
Check the units for (b): $\left(\frac{\frac{\mathrm{N}}{\mathrm{m}} \cdot \mathrm{m}^{2}}{\mathrm{~kg}}\right)^{\frac{1}{2}}=\left(\frac{\mathrm{N} \mathrm{m}}{\mathrm{kg}}\right)^{\frac{1}{2}}=\left(\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^{2}\right)^{\frac{1}{2}}=\frac{\mathrm{m}}{\mathrm{s}}$
The speed is less than the initial speed, as expected (because the spring slows down the cart).

## 10. Conceptualize the Problem

- The car is moving so it has kinetic energy.
- The spring does negative work on the car, lowering its kinetic energy and ultimately bringing it to a stop.
- The car does work on the spring, giving it elastic potential energy.
- The car is coasting horizontally, so there is no change in gravitational potential energy.
- Assume that friction can be ignored.
- The law of conservation of energy applies to this problem.


## Identify the Goal

The spring constant, $k$, of the spring

## Identify the Variables

| Known | Implied | Unknown |
| :--- | :---: | :--- |
| $m=150 \mathrm{~kg}$ | $v^{\prime}=0 \frac{\mathrm{~m}}{\mathrm{~s}}$ | $k$ |
| $v=6.0 \frac{\mathrm{~m}}{\mathrm{~s}}$ |  |  |
| $x=2.0 \mathrm{~m}$ |  |  |

## Develop a Strategy

Write the law of conservation of energy, including the energy quantities associated with the interaction.
Initially, the spring is not compressed, so the initial elastic potential energy is zero. After the interaction, the car stopped, so the kinetic energy is zero.

Substitute the values for energy listed by substituting the expressions for the energies.
Solve for the spring constant.
Substitute numerical values and solve.

The spring constant is about

$$
1.4 \times 10^{3} \frac{\mathrm{~N}}{\mathrm{~m}} .
$$

## Calculations

$E_{\mathrm{k}}^{\prime}+E_{\mathrm{e}}^{\prime}=E_{\mathrm{k}}+E_{\mathrm{e}}$
$E_{\mathrm{e}}=0 \mathrm{~J}$
$E_{\mathrm{k}}^{\prime}=0 \mathrm{~J}$
$0 \mathrm{~J}+E_{\mathrm{e}}^{\prime}=E_{\mathrm{k}}+0 \mathrm{~J}$
$E_{\mathrm{e}}^{\prime}=E_{\mathrm{k}}$
$\frac{1}{2} k x^{2}=\frac{1}{2} m v^{2}$
$k=\frac{m v^{2}}{x}$
$k=\frac{(150 \mathrm{~kg})\left(6.0 \frac{\mathrm{~m}}{\mathrm{~s}}\right)^{2}}{(2.0 \mathrm{~m})^{2}}$
$k=1350 \frac{\mathrm{~kg}}{\mathrm{~s}^{2}}$
$k \cong 1.4 \times 10^{3} \frac{\mathrm{~N}}{\mathrm{~m}}$

Validate the Solution
The units $\frac{\mathrm{kg}}{\mathrm{s}^{2}}$ are equivalent to $\frac{\mathrm{N}}{\mathrm{m}}$.
The spring constant is large and it is reasonable to be able to stop a 150 kg car.

## 11. Conceptualize the Problem

- The elastic potential energy of the bow increases as it is stretched.
- When the bow is released, the arrow will have kinetic energy.
- The problem does not involve a change of height, so there is no change in gravitational potential energy.
- The law of conservation of energy applies to this problem.


## Identify the Goal

The speed, $v$, of the arrow at the moment it leaves the bow.

## Identify the Variables

Known
$m=0.030 \mathrm{~kg}$
$x=45.0 \mathrm{~cm}$
$k=485 \mathrm{~N} / \mathrm{m}$

## Develop a Strategy

Write the law of conservation of energy, including the energy quantities associated with the interaction.

Initially, the bow is stretched and
the arrow has zero kinetic energy.
When the bow returns to its equilibrium position, the arrow's kinetic energy is a maximum.

Substitute the values for energy listed above.
Expand by substituting the
expressions for the energies.
Solve for the speed of the arrow.
Substitute numerical values and solve.

$v=0 \mathrm{~m} / \mathrm{s}$

Unknown
$v^{\prime}$

## Calculations

$E_{\mathrm{k}}^{\prime}+E_{\mathrm{e}}^{\prime}=E_{\mathrm{k}}+E_{\mathrm{e}}$

$$
E_{\mathrm{k}}=0 \mathrm{~J}
$$

$$
E_{\mathrm{e}}^{\prime}=0 \mathrm{~J}
$$

$$
0 \mathrm{~J}+E_{\mathrm{k}}^{\prime}=E_{\mathrm{e}}+0 \mathrm{~J}
$$

$$
E_{\mathrm{k}}^{\prime}=E_{\mathrm{e}}
$$

$$
\frac{1}{2} m v^{\prime 2}=\frac{1}{2} k x^{2}
$$

$$
v^{\prime}=\sqrt{\frac{k x^{2}}{m}}
$$

$$
v^{\prime}=\sqrt{\frac{485 \mathrm{~N} / \mathrm{m}(0.45 \mathrm{~m})^{2}}{0.030 \mathrm{~kg}}}
$$

$$
v^{\prime}=57.22 \mathrm{~m} / \mathrm{s}
$$

$$
v^{\prime} \cong 57 \mathrm{~m} / \mathrm{s}
$$

The speed of the arrow when it leaves the bow is $57 \mathrm{~m} / \mathrm{s}$.
Validate the Solution
Check the units: $\left(\frac{\frac{\mathrm{N}}{\mathrm{m}}(\mathrm{m})^{2}}{\mathrm{~kg}}\right)^{\frac{1}{2}}=\left(\frac{\mathrm{Nm}}{\mathrm{kg}}\right)^{\frac{1}{2}}=\left(\frac{\left(\frac{\mathrm{kgm}}{\mathrm{s}^{2}}\right) \mathrm{m}}{\mathrm{kg}}\right)^{\frac{1}{2}}=\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^{\frac{1}{2}}=\mathrm{m} / \mathrm{s}$
The result seems reasonable for a bow and arrow.

## 12. Conceptualize the Problem

- The elastic potential energy of the spring increases as it is stretched.
- The problem does not involve a change of height, so there is no change in gravitational potential energy.
- Energy loss due to friction can be neglected.
- When the spring is released, the mass will gain kinetic energy.
- The law of conservation of energy applies to this problem.


## Identify the Goal

(a) The maximum speed, $v$, of the mass.
(b) The speed, $v$, of the mass at 3.00 cm on either side of its equilibrium position.

## Identify the Variables

| Known | Implied | Unknown |
| :--- | :--- | :--- |
| $m=0.0250 \mathrm{~kg}$ | $v=0 \mathrm{~m} / \mathrm{s}$ | $v^{\prime}$ |
| $x=9.00 \mathrm{~cm}$ |  | $v^{\prime}($ when $x=3.00 \mathrm{~cm})$ |

$k=124 \mathrm{~N} / \mathrm{m}$

## Develop a Strategy

Write the law of conservation of

## Calculations

$E_{\mathrm{k}}^{\prime}+E_{\mathrm{e}}^{\prime}=E_{\mathrm{k}}+E_{\mathrm{e}}$ energy, including the energy quantities associated with the interaction.

Initially, the spring is stretched and $\quad E_{\mathrm{k}}=0 \mathrm{~J}$ the mass has zero kinetic energy.

When the spring returns to its

$$
\begin{aligned}
& E_{\mathrm{e}}^{\prime}=0 \mathrm{~J} \\
& 0 \mathrm{~J}+E_{\mathrm{k}}^{\prime}=E_{\mathrm{e}}+0 \mathrm{~J} \\
& E_{\mathrm{k}}^{\prime}=E_{\mathrm{e}}
\end{aligned}
$$

equilibrium position (where its potential energy is zero), the kinetic energy of the mass is a maximum.

Substitute the values for energy

$$
\frac{1}{2} m v^{\prime 2}=\frac{1}{2} k x^{2}
$$

listed above.
Expand by substituting the expressions for the energies.
Solve for the speed of the mass.
Substitute numerical values and solve.

$$
\begin{aligned}
& v^{\prime}=\sqrt{\frac{k x^{2}}{m}} \\
& v^{\prime}=\sqrt{\frac{124 \mathrm{~N} / \mathrm{m}(0.0900 \mathrm{~m})^{2}}{0.0250 \mathrm{~kg}}} \\
& v^{\prime}=6.3384 \mathrm{~m} / \mathrm{s} \\
& v^{\prime} \cong 6.34 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

(a) The maximum speed of the mass is $6.34 \mathrm{~m} / \mathrm{s}$.

Apply the law of conservation of energy to the situation. Let the initial position be the equilibrium position and the final position be 3.00 cm .
At the equilibrium position, the velocity is a maximum and the potential energy is zero.

Substitute values listed for the energies.

Substitute numerical values.

$$
\begin{aligned}
& E_{\mathrm{k}(\mathrm{x}=3.0 \mathrm{~cm})}^{\prime}+E_{\mathrm{e}(\mathrm{x}=3.0 \mathrm{~cm})}^{\prime}=E_{\mathrm{k}(x=\mathrm{eq} b \mathrm{~m})}+0 \\
& \frac{1}{2} m v^{\prime 2}+\frac{1}{2} k x^{2}=\frac{1}{2} m v^{2}+0 \\
& m v^{\prime 2}=m v^{2}-k x^{2} \\
& v^{\prime 2}=v^{2}-\frac{k}{m} x^{2}
\end{aligned}
$$

$$
E_{\mathrm{k}(x=3.0 \mathrm{~cm})}^{\prime}+E_{e(x=3.0 \mathrm{~cm})}^{\prime}=E_{\mathrm{k}(\mathrm{x}=\mathrm{eq} \mathrm{qm})}+E_{\mathrm{e}(\mathrm{x}=\mathrm{eq} \mathrm{q})}
$$

$$
E_{\mathrm{e}(\mathrm{x}=\mathrm{eqbm})}=0 \mathrm{~J}
$$

$$
v^{\prime}=\sqrt{(6.3384 \mathrm{~m} / \mathrm{s})^{2}-\frac{124 \mathrm{~N} / \mathrm{m}(0.0300 \mathrm{~m})^{2}}{0.0250 \mathrm{~kg}}}
$$

$$
v^{\prime}= \pm 5.9759 \mathrm{~m} / \mathrm{s}
$$

$$
v^{\prime} \cong 5.97 \mathrm{~m} / \mathrm{s}
$$

(b) The speed of the mass at 3.00 cm on either side of the equilibrium position is $5.97 \mathrm{~m} / \mathrm{s}$.

## Validate the Solution

The units work out to be $\mathrm{m} / \mathrm{s}$ as expected. The values for the speed are reasonable.

## 13. Conceptualize the Problem

- The elastic potential energy of the spring increases as it is stretched.
- The problem does not involve a change of height, so there is no change in gravitational potential energy.
- Energy loss due to friction can be neglected.
- When the spring is released, the mass will gain kinetic energy.
- The work done in stretching the spring is equivalent to the change in elastic potential energy.
- The law of conservation of energy applies to this problem.


## Identify the Goal

(a) The amplitude, $x$, of the motion of the mass.
(b) The work, $W$, done to stretch the spring to its maximum amplitude.

## Identify the Variables

| Known | Implied | Unknown |
| :--- | :--- | :--- |
| $m=0.150 \mathrm{~kg}$ | $v=0 \mathrm{~m} / \mathrm{s}$ | $x$ |
| $v^{\prime}=15.0 \mathrm{~m} / \mathrm{s}$ |  | $W$ |
| $k=215 \mathrm{~N} / \mathrm{m}$ |  |  |

## Develop a Strategy

Write the law of conservation of energy, including the energy quantities associated with the interaction.

When the spring is at the equilibrium position the potential energy is zero and the kinetic energy is a maximum.
When the displacement of the spring is a maximum, the velocity is zero.

## Calculations

$$
E_{\mathrm{k}(\mathrm{x}=\mathrm{max})}^{\prime}+E_{\mathrm{e}(\mathrm{x}=\mathrm{max})}^{\prime}=E_{\mathrm{k}(\mathrm{x}=\mathrm{eqbm})}+E_{\mathrm{e}(\mathrm{x}=\mathrm{eq} b \mathrm{~m})}
$$

Substitute the values for energy listed

$$
\begin{aligned}
& 0+E_{\mathrm{e}(\mathrm{x}=\max )}^{\prime}=E_{\mathrm{k}(x=\mathrm{eqbm})}+0 \\
& \frac{1}{2} k x^{2}=\frac{1}{2} m v^{2} \\
& x=\sqrt{\frac{m}{k} v^{2}} \\
& x=\sqrt{\frac{0.150 \mathrm{~kg}}{215 \mathrm{~N} / \mathrm{m}}(15.0 \mathrm{~m} / \mathrm{s})^{2}} \\
& x= \pm 0.3962 \mathrm{~m} \\
& x \cong 0.396 \mathrm{~m}
\end{aligned}
$$

(a) The amplitude of the motion of the mass is 0.396 m .

The work done to stretch the spring is the change in potential energy. The initial potential energy of the unstretched spring is zero.
Write the expression for the potential $\quad W=\frac{1}{2} k x^{2}$ energy at the maximum amplitude.

Substitute and solve.

$$
\begin{aligned}
W & =\frac{1}{2}(215 \mathrm{~N} / \mathrm{m})(0.3962 \mathrm{~m})^{2} \\
W & =16.8748 \mathrm{~N} \cdot \mathrm{~m} \\
W & \cong 16.9 \mathrm{~J}
\end{aligned}
$$

(b) The work done to stretch the spring to its maximum amplitude is 16.9 J .

Validate the Solution
Check the units for (a): $\left(\frac{\frac{\mathrm{N}}{\mathrm{m}}(\mathrm{m})^{2}}{\mathrm{~kg}}\right)^{\frac{1}{2}}=\left(\frac{\mathrm{kg}}{\mathrm{N} / \mathrm{m}}(\mathrm{m} / \mathrm{s})^{2}\right)^{\frac{1}{2}}=\left(\frac{\left(\frac{\mathrm{kgm}^{3}}{\mathrm{~s}^{2}}\right)}{\mathrm{kg} \mathrm{m} / \mathrm{s}^{2}}\right)^{\frac{1}{2}}=\left(\mathrm{m}^{2}\right)^{\frac{1}{2}}=\mathrm{m}$,
as required. The values seem reasonable.

## 14. Conceptualize the Problem

- The elastic potential energy of the spring increases as it is stretched.
- The problem does not involve a change of height, so there is no change in gravitational potential energy.
- Energy loss due to friction can be neglected.
- When the spring is released, the mass will gain kinetic energy.
- The work done in stretching the spring is equivalent to the change in elastic potential energy.
- The law of conservation of energy applies to this problem.


## Identify the Goal

(a) The mass, $m$, of the object.
(b) The amplitude, $x$, of the displacement of the spring.
(c) The object's position, $x^{\prime}$, when its speed was $5.00 \mathrm{~m} / \mathrm{s}$.

## Identify the Variables

| Known | Implied | Unknown |
| :--- | :--- | :--- |
| $v=14.6 \mathrm{~m} / \mathrm{s}$ | $v($ when $x=\max )=0 \mathrm{~m} / \mathrm{s}$ | $m$ |
| $v^{\prime}=5.0 \mathrm{~m} / \mathrm{s}$ |  | $x$ |
| $k=235 \mathrm{~N} / \mathrm{m}$ |  | $x^{\prime}$ |

## Develop a Strategy

Calculations
Before the mass can be found through the law of conservation of energy, the maximum amplitude that the spring is stretched can be found from the work done.

The work done to stretch the spring to its maximum amplitude is equal to the change in potential energy. (The unstretched spring has zero potential energy.)
Solve for the amplitude and substitute numerical values.

Apply the law of conservation of energy.

Note that at the maximum amplitude, the kinetic energy is zero, and at the equilibrium position, the elastic potential energy is zero.

Substitute the expressions for the energy.
(a) The mass of the object is 0.469 kg .
(b) Its amplitude was 0.652 m .

Apply the law of conservation of energy; consider the equilibrium position as the initial position and the position when the object has a speed of $5.00 \mathrm{~m} / \mathrm{s}$ as the final position.
The elastic potential energy at the equilibrium position, $E_{\mathrm{e}}=0 \mathrm{~J}$.

Expand the energy terms.
Solve for the position.

Substitute numerical values and solve.

$$
W=\frac{1}{2} k x^{2}
$$

$$
x=\sqrt{\frac{2 W}{k}}
$$

$$
x=\sqrt{\frac{2(50.0 \mathrm{~J})}{235 \mathrm{~N} / \mathrm{m}}}
$$

$$
x= \pm 0.65233 \mathrm{~m}
$$

$$
x \cong 0.652 \mathrm{~m}
$$

$$
E_{\mathrm{k}(\mathrm{x}=\max )}^{\prime}+E_{\mathrm{e}(\mathrm{x}=\max )}^{\prime}=E_{\mathrm{k}(\mathrm{x}=\mathrm{eq} b \mathrm{~m})}+E_{\mathrm{e}(\mathrm{x}=\mathrm{eq} b \mathrm{~m})}
$$

$$
E_{e(x=e q b m)}=0 \mathrm{~J}
$$

$$
E_{\mathrm{k}(\mathrm{x}=\max )}^{\prime}=0 \mathrm{~J}
$$

$$
0+E_{\mathrm{e}(x=\max )}^{\prime}=E_{\mathrm{k}(\mathrm{x}=\mathrm{eq} \mathrm{qm})}+0
$$

$$
\frac{1}{2} k x^{2}=\frac{1}{2} m v^{2}
$$

$$
m=k \frac{x^{2}}{v^{2}}
$$

$$
m=(235 \mathrm{~N} / \mathrm{m}) \frac{(0.65233 \mathrm{~m})^{2}}{(14.6 \mathrm{~m} / \mathrm{s})^{2}}
$$

$$
m=0.4691 \mathrm{~kg}
$$

$$
m \cong 0.469 \mathrm{~kg}
$$

$$
E_{\mathrm{k}}^{\prime}+E_{\mathrm{e}}^{\prime}=E_{\mathrm{k}}+E_{\mathrm{e}}
$$

## Practice Problem Solutions

## Student Textbook pages 298-299

## 15.Conceptualize the Problem

- Draw a sketch of the situation and identify the initial conditions.
- The person is falling, so he/she has kinetic energy and potential energy.
- By the law of conservation of energy, as the person falls, the gravitational potential energy is transformed into kinetic energy.
- Before the person lands on the net, the net has no elastic potential energy.
- The gravitational potential energy and kinetic energy of the person upon landing on the net are converted to elastic potential energy as the net is stretched and the person's gravitational potential energy and kinetic energy are reduced to zero.
- When the person comes to a stop (i.e. just touches the ground), he/she has no kinetic energy or gravitational potential energy. All of the energy is now stored in the net in the form of elastic potential energy.
- The person has no kinetic energy before dropping from the building, and when he/she just touches the ground.
- Since all of the motion is in a downward direction, define "down" as the positive direction for this problem.



## Identify the Goal

The spring constant, $k$, of the net

## Identify the Variables

Known
$h=8.00 \mathrm{~m}+1.40 \mathrm{~m}=9.40 \mathrm{~m}$
$x=1.40 \mathrm{~m}$
$m=70.0 \mathrm{~kg}$

## Develop a Strategy

Write the law of conservation of energy for the person for when he/she drops from the building to when the net just touches the ground.
Initially, the net is not compressed, so the initial elastic potential energy is zero. The velocity is zero before he/she drops and zero when he/she just touches the ground, so the kinetic energy is zero.

## Calculations

$E_{\mathrm{g}}^{\prime}+E_{\mathrm{k}}^{\prime}+E_{\mathrm{e}}^{\prime}=E_{\mathrm{g}}+E_{\mathrm{k}}+E_{\mathrm{e}}$
$E_{\mathrm{e}}=0 \mathrm{~J}$
$E_{\mathrm{k}}^{\prime}=E_{\mathrm{k}}=0 \mathrm{~J}$
$E_{\mathrm{g}}^{\prime}=0 \mathrm{~J}$

On the ground, the gravitational potential energy is zero.
Substitute the values for energy listed above.
Solve for the spring constant.
Substitute numerical values and solve.

$$
\begin{aligned}
& 0 \mathrm{~J}+0 \mathrm{~J}+E_{\mathrm{e}}^{\prime}=E_{\mathrm{g}}+0 \mathrm{~J}+0 \mathrm{~J} \\
& E_{\mathrm{e}}^{\prime}=E_{\mathrm{g}} \\
& \frac{1}{2} k x^{2}=m g h \\
& k=\frac{2 m g h}{x^{2}} \\
& k=\frac{2(70.0 \mathrm{~kg})\left(9.81 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}\right)(9.40 \mathrm{~m})}{(1.40 \mathrm{~m})^{2}} \\
& k=6586.71 \frac{\mathrm{~kg}}{\mathrm{~s}^{2}} \\
& k
\end{aligned}
$$

The spring constant is about
$6.59 \times 10^{3} \frac{\mathrm{~N}}{\mathrm{~m}}$.

## Validate the Solution

It is expected that the magnitude of the spring constant will be high because the net must be able to stop a person in a very short distance. The value $6.59 \times 10^{3} \frac{\mathrm{~N}}{\mathrm{~m}}$ seems reasonable.
Note that it is also possible to do this problem in two parts: (i) use conservation of energy (gravitational and kinetic) for the initial and final conditions of dropping from the building and striking the net, (ii) use conservation of energy (gravitational potential, kinetic and elastic potential) for the initial and final conditions of striking the net and just touching the ground. This method is equivalent, but slightly more involved because, as shown in the method above, the kinetic energy at the initial and final positions is zero.

## 16. Conceptualize the Problem

- Draw a sketch of the situation and identify the initial conditions.
- The block is falling, so it has kinetic energy and potential energy.
- By the law of conservation of energy, as the block falls, the gravitational potential energy is transformed into kinetic energy.
- Before the block hits the spring, the spring has no elastic potential energy.
- The gravitational potential energy and kinetic energy of the block upon hitting the spring are converted to elastic potential energy as the spring is compressed.
- At the maximum compression of the spring, the block's kinetic energy is zero.
- At the maximum compression of the spring, the block's gravitational potential energy can be defined as zero, which means its initial gravitational potential energy is defined by the initial height plus the maximum compression distance of the spring.
- Thus, when the block comes to a stop on the spring, all its gravitational potential and kinetic energy are now stored in the spring in the form of elastic potential energy.
- Since all of the motion is in a downward direction, define "down" as the positive direction for this problem.


## Identify the Goal

The maximum compression of the spring, $x$, when the block comes to rest
Identify the Variables

| Known | Implied | Unknown |
| :--- | :--- | :--- |
| $m=6.0 \mathrm{~kg}$ |  |  |
| $h=1.80 \mathrm{~m}$ | $g=9.81 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}$ | $x$ |
| $v=4.0 \frac{\mathrm{~m}}{\mathrm{~s}}$ | $v_{\mathrm{f}}=0 \frac{\mathrm{~m}}{\mathrm{~s}}$ |  |
| $k=2.000 \times 10^{3} \frac{\mathrm{~N}}{\mathrm{~m}}$ |  |  |

## Develop a Strategy

Write the law of conservation of energy for the block.
Initially, the spring is not compressed, so the initial elastic potential energy is zero.
Choose the lowest level of the spring (maximum compression of the spring) as the reference level for gravitational potential energy. The block comes to rest at the lowest point.
Substitute the initial and
final conditions into the above equation and substitute the individual expressions for the energies.
The change in height is the initial height, 1.80 m , plus the amount the spring is compressed, $x$.
The equation yields a quadratic equation.
Substitute values and use the quadratic formula to solve.

Choose the positive value of the quadratic formula.
The maximum compression of the spring will be about 0.42 m .

## Calculations

$E_{\mathrm{g}}^{\prime}+E_{\mathrm{k}}^{\prime}+E_{\mathrm{e}}^{\prime}=E_{\mathrm{g}}+E_{\mathrm{k}}+E_{\mathrm{e}}$

$$
E_{\mathrm{e}}=0 \mathrm{~J}
$$

$$
E_{\mathrm{g}}^{\prime}=0 \mathrm{~J}
$$

$$
E_{\mathrm{k}}^{\prime}=0 \mathrm{~J}
$$

$$
0 \mathrm{~J}+0 \mathrm{~J}+E_{\mathrm{e}}^{\prime}=E_{\mathrm{g}}+E_{\mathrm{k}}+0 \mathrm{~J}
$$

$$
E_{\mathrm{e}}^{\prime}=E_{\mathrm{g}}+E_{\mathrm{k}}
$$

$$
\frac{1}{2} k x^{2}=m g \Delta h+\frac{1}{2} m v^{2}
$$

$$
\frac{1}{2} k x^{2}=m g(1.80+x)+\frac{1}{2} m v^{2}
$$

$$
\frac{1}{2} k x^{2}-m g x-1.80 m g-\frac{1}{2} m v^{2}=0
$$

$$
\frac{1}{2}\left(2.000 \times 10^{3} \frac{\mathrm{~N}}{\mathrm{~m}}\right) x^{2}-(6.0 \mathrm{~kg})\left(9.81 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}\right) x
$$

$$
-1.80(6.0 \mathrm{~kg})\left(9.81 \frac{\mathrm{~m}}{\mathrm{~s}}\right)-\frac{1}{2}(6.0 \mathrm{~kg})\left(4.0 \frac{\mathrm{~m}}{\mathrm{~s}}\right)^{2}=0
$$

$$
\left(1.000 \times 10^{3}\right) x^{2}-58.86 x-153.95=0
$$

$$
x=\frac{58.86 \pm \sqrt{(-58.86)^{2}-4\left(1.000 \times 10^{3}\right)(-153.95)}}{2\left(1.000 \times 10^{3}\right)}
$$

$$
x=0.02943 \pm 0.3935 \mathrm{~m}
$$

$$
x \cong 0.42 \mathrm{~m}
$$

## Validate the Solution

Check the units: a term by term examination of the equation shows that the units for newtons are common to each term and can be divided out; each term then has the unit of metres, as required. The maximum compression of 0.42 m is a reasonable value for the given height and velocity of the block.

## 17. Conceptualize the Problem

- Draw a sketch of the situation and identify the initial conditions.
- At the bridge, and when the jumper's head just touches the water, the kinetic energy is zero.
- At the bridge, the jumper's gravitational potential energy is maximum.
- By falling, the jumper gains kinetic energy but loses gravitational potential energy.
- While stretching the bungee cord, kinetic energy and gravitational potential energy are converted into elastic potential energy.
- When the jumper's head just touches the water, the elastic potential energy of the bungee cord is a maximum and the kinetic energy and gravitational potential energy are zero.
- The amount that the cord can stretch is determined from the height of the bridge, the length of the unstretched cord and the height of the jumper.
- For part (b), draw a free-body diagram to determine the forces acting on the jumper: the restoring force of the cord is countered by the weight of the jumper.
- Define "up" as the positive direction for this problem.



## Identify the Goal

(a) the spring constant, $k$, for the bungee cord
(b) the acceleration, $a$, of the jumper at the bottom of the descent

## Identify the Variables

Known Implied Unknown
$\Delta h=h_{\mathrm{f}}-h_{\mathrm{i}}=0 \mathrm{~m}-22 \mathrm{~m}=-22.0 \mathrm{~m}$
$m=60.0 \mathrm{~kg}$
$g=-9.81 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}$
$h_{1}=12.2 \mathrm{~m}$
$h_{2}=1.8 \mathrm{~m}$
$x=8.0 \mathrm{~m}$ [The step to calculate " $x$ "
is in the "Calculation" below.]

Unknown
$k$

## Develop a Strategy

Write the law of conservation of energy for the jumper.
Initially, the cord is not compressed, so the initial elastic potential energy is zero.

Choose the lowest level of the cord (maximum stretching) as the reference level for gravitational potential energy.
The jumper is initially at rest and comes to rest at the lowest point.
Substitute the initial and final conditions into the above equation and substitute the

## Calculations

$E_{\mathrm{g}}^{\prime}+E_{\mathrm{k}}^{\prime}+E_{\mathrm{e}}^{\prime}=E_{\mathrm{g}}+E_{\mathrm{k}}+E_{\mathrm{e}}$

$$
E_{\mathrm{e}}=0 \mathrm{~J}
$$

$$
E_{\mathrm{g}}^{\prime}=0 \mathrm{~J}
$$

$$
\begin{aligned}
& E_{\mathrm{k}}^{\prime}=E_{\mathrm{k}}=0 \mathrm{~J} \\
& 0 \mathrm{~J}+0 \mathrm{~J}+E_{\mathrm{e}}^{\prime}=E_{\mathrm{g}}+0 \mathrm{~J}
\end{aligned}
$$

$$
E_{\mathrm{e}}^{\prime}=E_{\mathrm{g}}
$$

$$
\frac{1}{2} k x^{2}=m g \Delta h
$$

individual expressions for the energies.
The stretching distance of the cord is determined from

$$
\begin{aligned}
& x=h_{\mathrm{i}}-h_{1}-h_{2} \\
& x=22.0 \mathrm{~m}-12.2 \mathrm{~m}-1.8 \mathrm{~m}=8.0 \mathrm{~m}
\end{aligned}
$$

the height of the bridge, the unstretched length of the cord, and the jumper's height.
Substitute values and solve.

$$
\begin{aligned}
& k=\frac{2 m g \Delta h}{x^{2}} \\
& k=\frac{2(60.0 \mathrm{~kg})\left(-9.81 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}\right)(-22.0 \mathrm{~m})}{(8.0 \mathrm{~m})^{2}} \\
& k=404.66 \frac{\mathrm{~kg}}{\mathrm{~s}^{2}} \\
& k \cong 405 \frac{\mathrm{~N}}{\mathrm{~m}}
\end{aligned}
$$

(a) The spring constant of the bungee cord is about $405 \mathrm{~N} / \mathrm{m}$. Apply Newton's second

$$
\begin{aligned}
F_{\text {net }}= & F_{\mathrm{a}}+F_{\mathrm{g}}=m a \\
k x+m g & =m a \\
a & =\frac{k x+m g}{m} \\
a & =\frac{\left(404.66 \frac{\mathrm{~N}}{\mathrm{~m}}\right)(8.0 \mathrm{~m})+(60.0 \mathrm{~kg})\left(-9.81 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}\right)}{(60.0 \mathrm{~kg})} \\
& =44.145 \frac{\mathrm{~m}}{\mathrm{~s}^{2}} \\
& \cong 44.1 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}
\end{aligned}
$$

(b) The jumper accelerates $44.1 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}$ upwards.
Validate the Solution
In each case, the units are appropriate to the variable.

## Practice Problem Solutions

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## 18. Conceptualize the Problem

- Draw a sketch of the situation and identify the initial conditions.
- As the roller-coaster car moves down the track, most of the gravitational potential energy is converted into kinetic energy, but some is lost as heat due to friction.
- The law of conservation of total energy applies.
- Heat energy must be included as a final energy.


## Identify the Goal

The speed of the roller-coaster car at point $\mathrm{C}, v_{C}$

## Identify the Variables

| Known | Implied | Unknown |
| :--- | :--- | :--- |
| $h_{\mathrm{A}}=15.0 \mathrm{~m}$ | $g=9.81 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}$ | $v_{\mathrm{C}}$ |
| $h_{\mathrm{B}}=6.0 \mathrm{~m}$ |  |  |
| $h_{\mathrm{C}}=8.0 \mathrm{~m}$ |  |  |
| $v_{A}=4.0 \frac{\mathrm{~m}}{\mathrm{~s}}$ |  |  |
| $m=200.0 \mathrm{~kg}$ |  |  |
| $E_{\text {heat }}(\mathrm{A}$ to B$)=3.40 \times 10^{3} \mathrm{~J}$ |  |  |
| $E_{\text {heat }}$ ( to C $)=4.00 \times 10^{2} \mathrm{~J}$ |  |  |

## Develop a Strategy

Write the law of conservation of energy, including heat as a final energy form.
Substitute the expressions for the energies. Solve for the speed of the car at point $C$.

Substitute values and solve.

At point C , the roller-coaster car is travelling about $11 \frac{\mathrm{~m}}{\mathrm{~s}}$.

## Calculations

$$
\begin{aligned}
& E_{\mathrm{g}}^{\prime}+E_{\mathrm{k}}^{\prime}+E_{\text {heat }}=E_{\mathrm{g}}+E_{\mathrm{k}} \\
& \frac{1}{2} m v_{\mathrm{C}}^{2}+m g h_{\mathrm{C}}+E_{\text {heat }}=\frac{1}{2} m v_{\mathrm{A}}^{2}+m g h_{\mathrm{A}} \\
& \qquad v_{\mathrm{C}}^{2}=\frac{\frac{1}{2} m v_{\mathrm{A}}^{2}+m g h_{\mathrm{A}}-m g h_{\mathrm{C}}-E_{\text {heat }}}{\frac{1}{2} m} \\
& E_{\text {heat }}=3.40 \times 10^{3} \mathrm{~J}+4.00 \times 10^{2} \mathrm{~J} \\
& E_{\text {heat }}=3.80 \times 10^{2} \mathrm{~J} \\
& v_{\mathrm{C}}=\sqrt{\frac{1}{2}(200.0 \mathrm{~kg})\left(4.0 \frac{\mathrm{~m}}{2}\right)^{2}+(200.0 \mathrm{~kg})\left(9.81 \frac{\mathrm{~m}}{2}\right)(15.0 \mathrm{~m}-8.0 \mathrm{~m})-\left(3.80 \times 10^{3} \mathrm{~J}\right)} \\
& \frac{1}{2}(200.0 \mathrm{~kg}) \\
& v_{\mathrm{C}}=10.7396 \frac{\mathrm{~m}}{\mathrm{~s}} \\
& v_{\mathrm{C}} \cong 11 \frac{\mathrm{~m}}{\mathrm{~s}}
\end{aligned}
$$

## Validate the Solution

In each case, the units are appropriate to the variable. At point $C$, the car is higher than at B , so it is expected that the speed will be slower (compared to $v_{\mathrm{B}}=12.6 \frac{\mathrm{~m}}{\mathrm{~s}}$, from the Sample Problem), and it is.

## 19. Conceptualize the Problem

- Draw a sketch of the situation and identify the initial conditions.
- As the sled moves down the hill, most of the gravitational potential energy is converted into kinetic energy, but some is lost as heat due to friction.
- The work done on the snow is the same as the heat lost due to friction.
- The law of conservation of total energy applies.
- Heat energy must be included as a final energy.


## Identify the Goal

The speed of the sled at point $\mathrm{X}, v_{\mathrm{X}}$

## Identify the Variables

| Known | Implied | Unknown |
| :--- | :--- | :--- |
| $h_{\mathrm{A}}$ | $=12.0 \mathrm{~m}$ | $g=9.81 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}$ |
| $h_{\mathrm{X}}$ | $=3.0 \mathrm{~m}$ |  |
| $v_{\mathrm{A}}$ | $=8.0 \frac{\mathrm{~m}}{\mathrm{~s}}$ |  |
| $m=70.0 \mathrm{~kg}$ |  |  |
| $W=1.22 \times 10^{3} \mathrm{~J}$ |  |  |

## Develop a Strategy

Write the law of conservation of energy, including heat as a final energy form.
Substitute the expressions for the energies. Solve for the speed of the sled at point X .

## Calculations

$$
E_{\mathrm{g}}^{\prime}+E_{\mathrm{k}}^{\prime}+E_{\mathrm{heat}}=E_{\mathrm{g}}+E_{\mathrm{k}}
$$

$$
\begin{aligned}
& \frac{1}{2} m v_{\mathrm{X}}^{2}+m g h_{\mathrm{X}}+E_{\text {heat }}=\frac{1}{2} m v_{\mathrm{A}}^{2}+m g h_{\mathrm{A}} \\
& v_{\mathrm{X}}^{2}=\frac{\frac{1}{2} m v_{\mathrm{A}}^{2}+m g h_{\mathrm{A}}-m g h_{\mathrm{X}}-E_{\text {heat }}}{\frac{1}{2} m} \\
& v_{\mathrm{X}}=\sqrt{\frac{\frac{1}{2}(70.0 \mathrm{~kg})\left(8.0 \frac{\mathrm{~m}}{5}\right)^{2}+(70.0 \mathrm{~kg})\left(9.81 \frac{\mathrm{~m}}{\mathrm{~s}}\right)(12.0 \mathrm{~m}-3.0 \mathrm{~m})-\left(1.22 \times 10^{3} \mathrm{~J}\right)}{\frac{1}{2}(70.0 \mathrm{~kg})}} \\
& v_{\mathrm{X}}=14.3430 \frac{\mathrm{~m}}{\mathrm{~s}} \\
& v_{\mathrm{X}} \cong 14 \frac{\mathrm{~m}}{\mathrm{~s}}
\end{aligned}
$$

At point $X$, the sled has
a speed of about $14 \frac{\mathrm{~m}}{\mathrm{~s}}$.

## Validate the Solution

The sled increases its speed from $8.0 \frac{\mathrm{~m}}{\mathrm{~s}}$ to $14 \frac{\mathrm{~m}}{\mathrm{~s}}$ as it descends the hill, which seems reasonable.

## 20. Conceptualize the Problem

- Draw a sketch of the situation and identify the initial conditions.
- As the sled moves down the hill, most of the gravitational potential energy is converted into kinetic energy, but some is lost as heat due to friction.
- The work done on the snow is the same as the heat lost due to friction.
- The law of conservation of total energy applies.
- Heat energy must be included as a final energy.


## Identify the Goal

The work done on the snow, $W=E_{\text {heat, }}$, by the sled between points X and Y

## Identify the Variables

| Known | Implied | Unknown |
| :--- | :--- | :--- |
| $h_{\mathrm{A}}=12.0 \mathrm{~m}$ | $g=9.81 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}$ | $E_{\text {heat }}$ |
| $h_{\mathrm{X}}=3.00 \mathrm{~m}$ |  |  |
| $h_{\mathrm{Y}}=0.00 \mathrm{~m}$ |  |  |
| $v_{\mathrm{A}}=8.00 \frac{\mathrm{~m}}{\mathrm{~s}}$ |  |  |
| $v_{\mathrm{X}}=14.3 \frac{\mathrm{~m}}{\mathrm{~s}}$ |  |  |
| $v_{\mathrm{Y}}=15.6 \frac{\mathrm{~m}}{\mathrm{~s}}$ |  |  |
| $m=70.0 \mathrm{~kg}$ |  |  |

## Develop a Strategy

Write the law of conservation of energy, including heat as a final energy form.
Substitute the expressions for $\frac{1}{2} m v_{\mathrm{Y}}^{2}+m g h_{\mathrm{Y}}+E_{\text {heat }}=\frac{1}{2} m v_{\mathrm{X}}^{2}+m g h_{\mathrm{X}}$ the energies.
Substitute values and solve.

$$
\begin{aligned}
E_{\text {heat }}= & \frac{1}{2} m v_{\mathrm{x}}^{2}+m g h_{\mathrm{X}}-\frac{1}{2} m v_{\mathrm{Y}}^{2}-m g h_{\mathrm{Y}} \\
E_{\text {heat }}= & \frac{1}{2}(70.0 \mathrm{~kg})\left(14.3 \frac{\mathrm{~m}}{\mathrm{~s}}\right)^{2}+(70.0 \mathrm{~kg})\left(9.81 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}\right)(3.0 \mathrm{~m}) \\
& -\frac{1}{2}(70.0 \mathrm{~kg})\left(15.6 \frac{\mathrm{~m}}{\mathrm{~s}}\right)^{2}-0 \\
E_{\text {heat }}= & 739.75 \mathrm{~J} \\
E_{\text {heat }} \cong & 7.40 \times 10^{2} \mathrm{~J}
\end{aligned}
$$

## Calculations

$E_{\mathrm{g}}^{\prime}+E_{\mathrm{k}}^{\prime}+E_{\text {heat }}=E_{\mathrm{g}}+E_{\mathrm{k}}$

The sled does about
$7.40 \times 10^{2} \mathrm{~J}$ of work on the snow.

## Validate the Solution

Each term in the above equation has the units of joules, as required.

## 21. Frame the Problem

- The ball has gravitational potential energy, and no kinetic energy while on the shelf.
- The conservation of mechanical energy applies since frictional effects can be ignored.

PART A
Identify the Goal
The gravitational potential energy of the ball before it falls.

## Variables and Constants

| Known | Implied | Unknown |
| :--- | :--- | :--- |
| $m=0.50 \mathrm{~kg}$ | $g=9.81 \frac{\mathrm{~N}}{\mathrm{~kg}}$ | $E_{\mathrm{g}}$ |
| $\Delta h=2.3 \mathrm{~m}$ |  |  |

$\Delta h=2.3 \mathrm{~m}$

## Strategy

## Calculations

Use the formula for gravitational
$E_{\mathrm{g}}=m g \Delta h$ potential energy.
Substitute in the variables.
Simplify.
$E_{\mathrm{g}}=(0.50 \mathrm{~kg})\left(9.81 \frac{\mathrm{~N}}{\mathrm{~kg}}\right)(2.3 \mathrm{~m})$
$E_{\mathrm{g}}=11.28 \mathrm{~N} \cdot \mathrm{~m}$
$E_{\mathrm{g}}=11.28 \mathrm{~J}$
An $\mathrm{N} \cdot \mathrm{m}$ is equivalent to a J .
The gravitational potential energy of the ball while on the shelf is 11.29 J .
PART B
Identify the Goal
The speed of the ball as it strikes the floor.

## Strategy

Mechanical energy is conserved, thus gravitational potential energy at top equals kinetic energy at bottom.
Use the formula for kinetic energy to find the speed.
Substitute in the variables.

Simplify.
A $\sqrt{\frac{\mathrm{J}}{\mathrm{kg}}}$ is equivalent to a $\frac{\mathrm{m}}{\mathrm{s}}$.

## PART C

## Identify the Goal

The speed of the ball before you catch it.

## Strategy

Mechanical energy is conserved, thus its value is unchanged.
Use the formula for gravitational potential energy.
Substitute in the variables.
Simplify.
An $\mathrm{N} \cdot \mathrm{m}$ is equivalent to a J .
Use the formula for total mechanical energy to find kinetic energy.
Substitute in the variables.
Simplify.

## Calculations

$E_{\mathrm{k}(\text { bottom })}=E_{\mathrm{g}(\mathrm{top})}=11.28 \mathrm{~J}$
$v=\sqrt{\frac{2 E_{\mathrm{k}}}{\mathrm{m}}}$
$v=\sqrt{\frac{2(11.28 \mathrm{~J})}{0.50 \mathrm{~kg}}}$
$v=6.7\left(\sqrt{\frac{\mathrm{~J}}{\mathrm{~kg}}}\right)$
$v=6.7 \frac{\mathrm{~m}}{\mathrm{~s}}$

## Calculations

$E_{\mathrm{T}}=11.28 \mathrm{~J}$
$E_{\mathrm{g}}=m g \Delta h$
$E_{\mathrm{g}}=(0.50 \mathrm{~kg})\left(9.81 \frac{\mathrm{~N}}{\mathrm{~kg}}\right)(1.4 \mathrm{~m})$
$E_{\mathrm{g}}=6.88 \mathrm{~N} \cdot \mathrm{~m}$
$E_{\mathrm{g}}=6.88 \mathrm{~J}$
$E_{\mathrm{k}}=E_{\mathrm{T}}-E_{\mathrm{g}}$
$E_{\mathrm{k}}=11.28 \mathrm{~J}-6.88 \mathrm{~J}$
$E_{\mathrm{k}}=4.4 \mathrm{~J}$

Use the formula for kinetic energy to $\quad v=\sqrt{\frac{2 E_{\mathrm{k}}}{\mathrm{m}}}$
find the speed.
find the speed.
Substitute in the variables. $\quad v=\sqrt{\frac{2(4.4 \mathrm{~J})}{0.50 \mathrm{~kg}}}$
Simplify. $v=4.2\left(\sqrt{\frac{\mathrm{~J}}{\mathrm{~kg}}}\right)$
A $\sqrt{\frac{\mathrm{J}}{\mathrm{kg}}}$ is equivalent to an $\frac{\mathrm{m}}{\mathrm{s}}$.
$v=4.2 \frac{\mathrm{~m}}{\mathrm{~s}}$
The ball is still moving at $4.2 \mathrm{~m} / \mathrm{s}$ when you catch it.

## Validate

The mechanical energy of the ball was conserved as it fell and bounced back up again. The unit for gravitational potential energy, kinetic energy, and mechanical energy is the J. The unit for speed is the $\mathrm{m} / \mathrm{s}$.

## 22. Frame the Problem

- The force of friction does negative work on the bullet as it passes through the wood.
- The kinetic energy of the bullet decreases as it passes through the wood.
- The work-kinetic energy theorem applies.
- The force of friction can be found from the work-kinetic energy theorem.


## PART I

## Identify the Goal

The work done by the force of friction on the bullet as it passes through the wood.
Variables and Constants

| Known | Unknown |
| :--- | :--- |
| $m=2.0 \times 10^{-3} \mathrm{~kg}$ | $E_{\mathrm{k}_{1}}$ |
| $v_{1}=87 \mathrm{~m} / \mathrm{s}$ | $E_{\mathrm{k}_{2}}$ |
| $v_{2}=12 \mathrm{~m} / \mathrm{s}$ | $W$ |
| $\Delta d=4.0 \times 10^{-2} \mathrm{~m}$ | $F$ |

## Strategy

Use the kinetic energy formula to find the initial kinetic energy.
Substitute in the variables.
Simplify.
A $\mathrm{kg} \cdot \frac{\mathrm{m}^{2}}{\mathrm{~s}^{2}}$ is equivalent to a J .
Use the kinetic energy formula to find the final kinetic energy.
Substitute in the variables.

## Calculations

$E_{\mathrm{k}_{1}}=\frac{1}{2} m v^{2}$

Simplify.
A $\mathrm{kg} \cdot \frac{\mathrm{m}^{2}}{\mathrm{~s}^{2}}$ is equivalent to a J .
$E_{\mathrm{k}_{1}}=\frac{1}{2}\left(2.0 \times 10^{-3} \mathrm{~kg}\right)\left(87 \frac{\mathrm{~m}}{\mathrm{~s}}\right)^{2}$
$E_{\mathrm{k}_{1}}=7.569 \mathrm{~kg} \cdot \frac{\mathrm{~m}^{2}}{\mathrm{~s}^{2}}$
$E_{\mathrm{k}_{1}}=7.569 \mathrm{~J}$
$E_{\mathrm{k}_{2}}=\frac{1}{2} m v^{2}$

Use the work-kinetic energy theorem
to find the work done by friction.
Substitute in the variables.
$E_{\mathrm{k}_{2}}=\frac{1}{2}\left(2.0 \times 10^{-3} \mathrm{~kg}\right)\left(12 \frac{\mathrm{~m}}{\mathrm{~s}}\right)^{2}$
$E_{\mathrm{k}_{2}}=0.144 \mathrm{~kg} \cdot \frac{\mathrm{~m}^{2}}{\mathrm{~s}^{2}}$
$E_{\mathrm{k}_{2}}=0.144 \mathrm{~J}$
$W=E_{\mathrm{k}_{2}}-E_{\mathrm{k}_{1}}$

Simplify.
$W=0.144 \mathrm{~J}-7.569 \mathrm{~J}$

The work done on the bullet by the force of friction was -7.4 J .

PART II
Strategy
Use the formula for work to find the force of friction.
Substitute in the variables.

## Calculations

$F=\frac{W}{\Delta d}$

Simplify.
$F=\frac{-7.425 \mathrm{~J}}{4.0 \times 10^{-2} \mathrm{~m}}$
$\mathrm{A} \frac{\mathrm{J}}{\mathrm{m}}$ is equivalent to an m .
$F=-1.9 \times 10^{-2} \frac{\mathrm{~J}}{\mathrm{~m}}$
$F=-1.9 \times 10^{-2} \mathrm{~N}$
The force of friction on the bullet was $1.9 \times 10^{-2} \mathrm{~N}$ backwards.

## Validate

The bullet slowed down as it passed through the wood and its kinetic energy decreased. Negative work was done on the bullet by the force of friction, thus the work done and force of friction were both negative in value. The unit for work is the J and the unit for force is the N .

## 23. Frame the Problem

- The conservation of mechanical energy can be used to find the maximum possible speed of the roller coaster.
- The roller coaster's kinetic energy at the top of the track is assumed to be 0 J .
- The roller coaster's gravitational potential energy at the bottom of the track is 0 J .
- From the conservation of mechanical energy, the roller coaster's gravitational potential energy at the top is equal to its kinetic energy at the bottom of the track.
- Comparing the actual mechanical energy to the maximum possible mechanical energy will indicate the percentage of energy lost due to friction.


## PART I

## Identify the Goal

The maximum possible speed of the roller coaster.
Variables and Constants

| Known | Implied |
| :--- | :--- |
| $\Delta h=94.5 \mathrm{~m}$ | $E_{\mathrm{k} \text { (top) }}=0 \mathrm{~J}$ |
| $v=41.1 \mathrm{~m} / \mathrm{s}$ | $E_{\mathrm{g}(\text { bottom })}=0 \mathrm{~J}$ |
|  | $g=9.81 \frac{\mathrm{~N}}{\mathrm{~kg}}$ |

## Strategy

From the conservation of mechanical energy, the total mechanical at the bottom is equal to the total mechanical energy at the top.
Total mechanical energy is the sum of gravitational potential energy and kinetic energy.
Substitute in the variables for $E_{\mathrm{g}}$ and $E_{\mathrm{k}}$. Simplify.
Use the formulas for kinetic energy
and gravitational potential energy.
Rearrange to solve for the speed.

## Unknown

maximum $v$
maximum $E_{\mathrm{k}}$
actual $E_{\mathrm{k}}$
maximum $E_{\mathrm{T}}$
actual $E_{\mathrm{T}}$

## Calculations

$E_{\mathrm{T} \text { (bottom) }}=E_{\mathrm{T} \text { (top) }}$
$E_{\mathrm{g}(\mathrm{bottom})}+E_{\mathrm{k}(\text { bottom })}=E_{\mathrm{g}(\text { top })}+E_{\mathrm{k}(\text { top })}$
$0 \mathrm{~J}+E_{\mathrm{k}(\text { bottom })}=E_{\mathrm{g} \text { (top) }}+0 \mathrm{~J}$
$E_{\mathrm{k}(\text { bottom })}=E_{\mathrm{g}(\text { top })}$
$\frac{1}{2} m v^{2}=m g \Delta h$
$v=\sqrt{\frac{2 m g \Delta h}{m}}$

Cancel out the mass.
Substitute in the variables.
Simplify.
A $\left(\sqrt{\frac{\mathrm{N}}{\mathrm{kg}} \cdot \mathrm{m}}\right)$ is equivalent to an $\frac{\mathrm{m}}{\mathrm{s}}$.
$v=\sqrt{2 g \Delta h}$
$v=\sqrt{2\left(9.81 \frac{\mathrm{~N}}{\mathrm{~kg}}\right)(94.5 \mathrm{~m})}$
$v=43.1\left(\sqrt{\frac{\mathrm{~N}}{\mathrm{~kg} \cdot \mathrm{~m}}}\right)$
$v=43.1 \frac{\mathrm{~m}}{\mathrm{~s}}$

The maximum possible speed of the roller coaster is $43.1 \mathrm{~m} / \mathrm{s}$.

## PART II

## Identify the Goal

The percentage of total mechanical energy lost as heat.

## Strategy

## Calculations

Comparing the actual and maximum values of the kinetic energy is equivalent to a comparison of the actual and maximum values of the total mechanical energy. This is true since the gravitational potential energy at the bottom will be 0 J in both the actual and maximum cases.
Use the formula for kinetic energy in this expression.
Cancel out the $\left(\frac{1}{2} m\right)$.
Substitute in the variables.
Simplify.
Convert to a percentage.
The roller coaster actually has $90.9 \%$ of the maximum possible total mechanical energy.
Subtract this percentage from $100 \% \quad 100 \%-90.9 \%=9.1 \%$ to find the amount of energy lost.
The roller coaster lost $9.1 \%$ of its total mechanical energy due to friction.

## Validate

- The maximum possible speed of the roller coaster would be obtained only if there were no friction acting. Since there was friction in this case, the actual speed was less than the maximum possible speed.
- The unit for speed is the $\mathrm{m} / \mathrm{s}$.


## 24. Frame the Problem

- The child's kinetic energy while at the top of the slide is 0 J since she starts from rest.
- The child's gravitational potential energy while at the top of the slide can be found by using the formula for gravitational potential energy.
- The height of the slide can be found by using its length and the angle in the sine function.
- Since friction is acting on the child, her mechanical energy is not conserved.
- The amount of mechanical energy lost is equal to the amount of work done by the force of friction on the child.
- The formula for work can be used to find the force of friction.


## Identify the Goal

The force of friction exerted on the child by the slide.

## Variables and Constants

| Known | Implied | Unknown |
| :--- | :--- | :--- |
| $m=15 \mathrm{~kg}$ | $g=9.81 \frac{\mathrm{~N}}{\mathrm{~kg}}$ | $E_{\mathrm{g} \text { (top) }}$ |
| $L=4.0 \mathrm{~m}$ | $E_{\mathrm{k} \text { (top) }}=0 \mathrm{~J}$ | $E_{\mathrm{k}}$ |
| $\Theta=40^{\circ}$ | $E_{\mathrm{g} \text { (botom) }}=0 \mathrm{~J}$ | $E_{\mathrm{T}}$ |
| $v_{\text {(top) }}=0 \mathrm{~m} / \mathrm{s}$ |  |  |

## Strategy

Use the sine function to find the height of the slide.

Substitute in the variables.
Simplify.
Find the gravitational potential energy of the child while at the top of the slide.
Substitute in the variables.
Simplify.
An $\mathrm{N} \cdot \mathrm{m}$ is equivalent to a J .
Find the total mechanical energy at the top.

Substitute in the variables.
Simplify.
Find the child's kinetic energy while at the bottom of the slide.
Substitute in the variables.
Simplify.
A $\mathrm{kg} \cdot \frac{\mathrm{m}^{2}}{\mathrm{~s}^{2}}$ is equivalent to a J.
Find the actual total mechanical energy of the child while at the bottom of the slide.

Substitute in the variables.
Simplify.
Find the difference between the value of the total mechanical energy at the top and the bottom.

Substitute in the variables.
Simplify.
From the work-energy theorem, the work done is equal to the change in total mechanical energy.
Use the formula for work to find the force.
Substitute in the variables.
Simplify.
$E_{\mathrm{T}}=0 \mathrm{~J}+78.6 \mathrm{~J}$
$E_{\mathrm{T}}=78.6 \mathrm{~J}$
$\Delta E_{\mathrm{T}}=E_{\mathrm{T} \text { (bottom) }}-E_{\mathrm{T} \text { (top) }}$
$\Delta E_{\mathrm{T}}=78.6 \mathrm{~J}-378.18 \mathrm{~J}$
$\Delta E_{\mathrm{T}}=-299.58 \mathrm{~J}$
$W=\Delta E_{\mathrm{T}}=-299.58 \mathrm{~J}$

## Calculations

$\Delta h=L \cdot \sin \Theta$
$\Delta h=(4.0 \mathrm{~m})\left(\sin 40^{\circ}\right)$
$\Delta h=2.57 \mathrm{~m}$
$E_{\mathrm{g}}=m g \Delta h$
$E_{\mathrm{g}}=(15 \mathrm{~kg})\left(9.81 \frac{\mathrm{~N}}{\mathrm{~kg}}\right)(2.57 \mathrm{~m})$
$E_{\mathrm{g}}=378.18 \mathrm{~N} \cdot \mathrm{~m}$
$E_{\mathrm{g}}=378.18 \mathrm{~J}$
$E_{\mathrm{T}}=E_{\mathrm{g}}+E_{\mathrm{k}}$
$E_{\mathrm{T}}=378.18 \mathrm{~J}+0 \mathrm{~J}$
$E_{\mathrm{T}}=378.18 \mathrm{~J}$
$E_{\mathrm{k}}=\frac{1}{2} m v^{2}$
$E_{\mathrm{k}}=\frac{1}{2}(15 \mathrm{~kg})\left(3.2 \frac{\mathrm{~m}}{\mathrm{~s}}\right)^{2}$
$E_{\mathrm{k}}=78.6 \mathrm{~kg} \cdot \frac{\mathrm{~m}^{2}}{\mathrm{~s}^{2}}$
$E_{\mathrm{k}}=78.6 \mathrm{~J}$
$E_{\mathrm{T}}=E_{\mathrm{g}}+E_{\mathrm{k}}$
$F=\frac{W}{\Delta d}$
$F=\frac{-299.58 \mathrm{~J}}{4.0 \mathrm{~m}}$
$F=-74.895 \frac{\mathrm{~J}}{\mathrm{~m}}$

A $\frac{\mathrm{J}}{\mathrm{m}}$ is equivalent to an $\mathrm{N} . \quad F=-74.895 \mathrm{~N}$
The force of friction of the child is 75 N .
Validate

- The force of friction is exerted backwards on the child and thus does negative work on her. This negative work reduces her total mechanical energy, transforming some of it into thermal energy.


## Practice Problem Solutions

## Student Textbook page 315

## 25. Conceptualize the Problem

- Make a sketch of the momentum vectors representing conditions just before and just after the collision.
- Before the collision, both skaters are moving and therefore each has momentum.
- At the instant of the collision, momentum is conserved (that is, the momentum before the collision is the same as the momentum after the collision).
- After the collision, the two skaters move as one mass, with the same velocity.
- Let the direction of the skaters be forward.


## Identify the Goal

The velocity, $\vec{v}_{\text {CH }}^{\prime}$, of the combined skaters immediately after the collision

## Identify the Variables

| Known | Unknown |
| :--- | :---: |
| $m_{\mathrm{C}}$ | $=72 \mathrm{~kg}$ |
| $m_{\mathrm{H}}$ | $=47 \mathrm{~kg}$ |
| $\overrightarrow{v_{\mathrm{C}}}$ | $=3.1 \mathrm{~m} / \mathrm{s}[$ forward $]$ |
| $\vec{v}_{\mathrm{H}}$ | $=2.2 \mathrm{~m} / \mathrm{s}[$ forward $]$ |

## Develop a Strategy

Apply the law of conservation of momentum.

After the collision, the two masses act as one, with one velocity.
Rewrite the equation to show this condition, and solve for $\vec{v}_{\mathrm{CH}}^{\prime}$.

## Calculations

$$
\begin{aligned}
& m_{\mathrm{C}} \vec{v}_{\mathrm{V}}+m_{\mathrm{H}} \vec{v}_{\mathrm{H}}=m_{\mathrm{C}} \vec{v}_{\mathrm{C}}^{\prime}+m_{\mathrm{H}} \vec{v}_{\mathrm{v}}^{\prime} \\
& m_{\mathrm{C}} \vec{v}_{\mathrm{C}}+m_{\mathrm{H}} \vec{v}_{\mathrm{H}}=\left(m_{\mathrm{C}}+m_{\mathrm{H}}\right) \vec{v}_{\mathrm{C}}^{\prime}
\end{aligned}
$$

Substitute values and solve.

$$
\begin{aligned}
& \vec{v}_{\text {CH }}^{\prime}=\frac{(72 \mathrm{~kg})(3.1 \mathrm{~m} / \mathrm{s})[\text { forward }]+(47 \mathrm{~kg})(2.2 \mathrm{~m} / \mathrm{s})[\text { forward }]}{(72 \mathrm{~kg}+47 \mathrm{~kg})} \\
& \vec{v}_{\mathrm{CH}}^{\prime}=2.7445 \mathrm{~m} / \mathrm{s}[\text { forward }] \\
& \vec{v}_{\mathrm{CH}}^{\prime} \cong 2.7 \mathrm{~m} / \mathrm{s}[\text { forward }]
\end{aligned}
$$

The two skaters have a velocity of $2.7 \mathrm{~m} / \mathrm{s}[$ forward] .

## Validate the Solution

Both skaters are moving before the collision and it is expected that due to the collision, the slower skater will move faster and the faster skater will move slower afterwards, and this is observed.

## 26. Conceptualize the Problem

- Make a sketch of the momentum vectors representing conditions just before and just after the collision.
- Before the collision, both cars are moving and therefore each has momentum.
- At the instant of the collision, momentum is conserved (that is, the momentum before the collision is the same as the momentum after the collision).
- After the collision, the two cars move as one mass, with the same velocity.
- Since the motion is all in one dimension (i.e. the horizontal dimension), use plus and minus to denote direction. According to the information, car A is travelling in the positive direction. Let this be the forward direction.


## Identify the Goal

The velocity, $\vec{v}_{\mathrm{AB}}^{\prime}$, of the combined cars immediately after the collision

## Identify the Variables

| Known | Unknown |
| :--- | :---: |
| $m_{\mathrm{A}}$ | $=375 \mathrm{~kg}$ |
| $m_{\mathrm{B}}$ | $=422 \mathrm{~kg}$ |
| $\overrightarrow{v_{\mathrm{A}}}$ | $=1.8 \mathrm{~m} / \mathrm{s}$ [forward] |
| $\overrightarrow{v_{\mathrm{B}}}$ | $=-1.4 \mathrm{~m} / \mathrm{s}[$ forward $]$ |

## Develop a Strategy

Apply the law of conservation of momentum.

After the collision, the two masses act as one, with one velocity.
Rewrite the equation to show this condition, and solve for $\vec{v}_{A B}^{\prime}$

## Calculations

$$
\begin{aligned}
& m_{\mathrm{A}} \vec{v}_{\mathrm{A}}+m_{\mathrm{B}} \vec{v}_{\mathrm{B}}=m_{\mathrm{A}} \vec{v}_{\mathrm{A}}^{\prime}+m_{\mathrm{B}} \vec{v}_{\mathrm{B}}^{\prime} \\
& m_{\mathrm{A}} \vec{v}_{\mathrm{A}}+m_{\mathrm{B}} \vec{v}_{\mathrm{B}}=\left(m_{\mathrm{A}}+m_{\mathrm{B}}\right) \vec{v}_{\mathrm{A}}^{\prime}
\end{aligned}
$$

$$
\vec{v}_{\mathrm{AB}}^{\prime}=\frac{m_{\mathrm{A}} \vec{v}_{\mathrm{A}}+m_{\overrightarrow{\mathrm{B}}} \overrightarrow{\mathrm{~V}}_{\mathrm{B}}}{\left(m_{\mathrm{A}}+m_{\mathrm{B}}\right)}
$$

Substitute values and

$$
\begin{aligned}
& \vec{v}_{\mathrm{AB}}^{\prime}=\frac{(375 \mathrm{~kg})(1.8 \mathrm{~m} / \mathrm{s})[\text { forward }]+(422 \mathrm{~kg})(-1.4 \mathrm{~m} / \mathrm{s})[\text { forward }]}{(375 \mathrm{~kg}+422 \mathrm{~kg})} \\
& \vec{v}_{\mathrm{AB}}^{\prime}=0.1056 \mathrm{~m} / \mathrm{s}[\text { forward }] \\
& \vec{v}_{\mathrm{AB}}^{\prime} \cong 0.11 \mathrm{~m} / \mathrm{s}[\text { forward }]
\end{aligned}
$$

The two cars have a velocity of $0.11 \mathrm{~m} / \mathrm{s}$, in the direction of car A.

## Validate the Solution

The magnitude of the momentum of car $\mathrm{A}, \mathrm{m}_{\mathrm{A}} \mathrm{v}_{\mathrm{A}}=(375 \mathrm{~kg})(1.8 \mathrm{~m} / \mathrm{s})=675 \mathrm{~kg} \mathrm{~m} / \mathrm{s}$, is greater than the magnitude of the momentum of car $\mathrm{B}, \mathrm{m}_{\mathrm{B}} \mathrm{v}_{\mathrm{B}}=(422 \mathrm{~kg})(1.4 \mathrm{~m} / \mathrm{s})$ $=591 \mathrm{~kg} \mathrm{~m} / \mathrm{s}$, which is travelling in the opposite direction, so it is expected that the combined mass will move in the same direction as car A , and it does.

## Practice Problem Solutions

## Student Textbook page 317

## 27. Conceptualize the Problem

- Make a sketch of the momentum vectors representing conditions just before and just after the collision (i.e. the firing of the cannon).
- Before the cannon is fired, neither the cannon nor the cannon ball are moving, therefore, the momentum of each is zero.
- At the instant the cannon is fired, momentum is conserved (that is, the momentum before the collision is the same as the momentum after the collision).
- Because the cannon ball moves north, in order to conserve momentum, the cannon will recoil to the south (that is, in the opposite direction).


## Identify the Goal

The velocity, $\vec{v}_{A}^{\prime}$, of the cannon immediately after it is fired

## Identify the Variables

Known Unknown

$$
\begin{array}{ll}
m_{\mathrm{A}}=1385 \mathrm{~kg} & \overrightarrow{v_{\mathrm{A}}}=0 \mathrm{~m} / \mathrm{s} \\
m_{\mathrm{B}}=58.5 \mathrm{~kg} & \overrightarrow{v_{\mathrm{B}}}=0 \mathrm{~m} / \mathrm{s} \\
\vec{v}_{\mathrm{B}}^{\prime}=49.8 \mathrm{~m} / \mathrm{s}[\mathrm{~N}]
\end{array}
$$

## Develop a Strategy

Apply the law of conservation of momentum.

## Calculations

$$
\begin{aligned}
m_{\mathrm{A}} \overrightarrow{v_{\mathrm{A}}}+m_{\mathrm{B}} \overrightarrow{v_{\mathrm{B}}} & =m_{\mathrm{A}} \overrightarrow{v_{\mathrm{A}}^{\prime}}+m_{\mathrm{B}} \overrightarrow{v_{\mathrm{B}}^{\prime}} \\
0+0 & =m_{\mathrm{A}}{\overrightarrow{v_{\mathrm{A}}^{\prime}}}^{\prime}+m_{\mathrm{B}} \overrightarrow{v_{\mathrm{B}}^{\prime}} \\
\vec{v}_{\mathrm{A}}^{\prime} & =-\frac{m_{\mathrm{B}} \overrightarrow{v_{\mathrm{B}}^{\prime}}}{m_{\mathrm{A}}}
\end{aligned}
$$

Substitute values and solve.

$$
\begin{aligned}
\vec{v}_{\mathrm{A}}^{\prime} & =-\frac{(58.5 \mathrm{~kg})(49.8 \mathrm{~m} / \mathrm{s})[\mathrm{N}]}{1385 \mathrm{~kg}} \\
\vec{v}_{\mathrm{A}}^{\prime} & =-2.103 \mathrm{~m} / \mathrm{s}[\mathrm{~N}] \\
\vec{v}_{\mathrm{A}}^{\prime} & \cong 2.10 \mathrm{~m} / \mathrm{s}[\mathrm{~S}]
\end{aligned}
$$

The cannon moves $2.10 \mathrm{~m} / \mathrm{s}[\mathrm{S}]$ after it is fired.

## Validate the Solution

The cannon ball has a relatively low mass and high velocity (after being fired), so it is expected that the cannon, which has a much higher mass, will recoil with a low velocity, and it does.

## 28. Conceptualize the Problem

- Make a sketch of the momentum vectors representing conditions just before and just after the collision (i.e. the throwing of the rock).
- Before the rock is thrown, neither the rock nor the person are moving, therefore, the momentum of each is zero.
- At the instant the rock is thrown, momentum is conserved (that is, the momentum before the collision is the same as the momentum after the collision).
- Because the rock moves west, in order to conserve momentum, the person will recoil to the east (that is, in the opposite direction).


## Identify the Goal

The velocity, $\vec{v}_{A}^{\prime}$, of the person immediately after the rock is thrown

## Identify the Variables

| Known |  |
| :--- | :--- |
| $m_{\mathrm{A}}=57 \mathrm{~kg}$ | $\overrightarrow{v_{\mathrm{A}}}=0 \mathrm{~m} / \mathrm{s}$ |
| $m_{\mathrm{B}}=1.7 \mathrm{~kg}$ | $\overrightarrow{v_{\mathrm{B}}}=0 \mathrm{~m} / \mathrm{s}$ |
| $\overrightarrow{v_{\mathrm{B}}^{\prime}}=3.8 \mathrm{~m} / \mathrm{s}[\mathrm{W}]$ |  |

## Develop a Strategy

Apply the law of conservation of momentum.

Substitute values and solve.

## Calculations

$$
\begin{aligned}
m_{\mathrm{A}} \overrightarrow{v_{\mathrm{A}}}+m_{\mathrm{B}} \overrightarrow{v_{\mathrm{B}}} & =m_{\mathrm{A}} \vec{v}_{\mathrm{A}}^{\prime}+m_{\mathrm{B}}{\overrightarrow{v_{\mathrm{B}}^{\prime}}}_{0+0}=m_{\mathrm{A}} \vec{v}_{\mathrm{A}}^{\prime}+m_{\mathrm{B}} \overrightarrow{v_{\mathrm{B}}^{\prime}} \\
\vec{v}_{\mathrm{A}}^{\prime} & =-\frac{m_{\mathrm{B}} \overrightarrow{v_{\mathrm{B}}^{\prime}}}{m_{\mathrm{A}}}
\end{aligned}
$$

$$
\begin{aligned}
& \vec{v}_{\mathrm{A}}^{\prime}=-\frac{(1.7 \mathrm{~kg})(3.8 \mathrm{~m} / \mathrm{s})[\mathrm{W}]}{57 \mathrm{~kg}} \\
& \vec{v}_{\mathrm{A}}^{\prime}=-0.1133 \mathrm{~m} / \mathrm{s}[\mathrm{~W}] \\
& \vec{v}_{\mathrm{A}}^{\prime} \cong 0.11 \mathrm{~m} / \mathrm{s}[\mathrm{E}]
\end{aligned}
$$

The person moves $0.11 \mathrm{~m} / \mathrm{s}[\mathrm{E}]$ after the rock is thrown.
Validate the Solution
The rock has a relatively low mass and high velocity (after being thrown), so it is expected that the person, who has a much higher mass, will recoil with a low velocity, and he/she does.

## 29. Conceptualize the Problem

- Make a sketch of the momentum vectors representing conditions just before and just after the collision (i.e. the emission of the alpha particle).
- Before the alpha particle is emitted, the uranium atom is stationary (and it contains the alpha particle), therefore, the momentum of each is zero.
- At the instant the alpha particle is emitted, momentum is conserved (that is, the momentum before the collision is the same as the momentum after the collision).
- The mass of the uranium atom after the alpha particle is emitted is lower than before, by an amount equal to the mass of the alpha particle.
- Because the alpha particle moves in the positive direction, in order to conserve momentum, the uranium atom will recoil in the negative direction.


## Identify the Goal

The velocity, $\vec{v}_{\mathrm{A}}^{\prime}$, of the uranium atom immediately after the alpha particle is emitted

## Identify the Variables

## Known

$$
\begin{aligned}
& m_{\mathrm{A}}=3.95 \times 10^{-25} \mathrm{~kg} \quad \overrightarrow{v_{\mathrm{A}}}=0 \mathrm{~m} / \mathrm{s} \\
& m_{\mathrm{B}}=6.64 \times 10^{-27} \mathrm{~kg} \quad \overrightarrow{v_{\mathrm{B}}}=0 \mathrm{~m} / \mathrm{s} \\
& \vec{v}_{\mathrm{B}}^{\prime}=1.42 \times 10^{4} \mathrm{~m} / \mathrm{s}[\text { forward }]
\end{aligned}
$$

Unknown
$\vec{v}_{\mathrm{A}}^{\prime}$

## Develop a Strategy

Apply the law of conservation of momentum.

## Calculations

$$
\begin{aligned}
m_{\mathrm{A}} \overrightarrow{v_{\mathrm{A}}}+m_{\mathrm{B}} \overrightarrow{v_{\mathrm{B}}} & =m_{\mathrm{A}}{\overrightarrow{v_{\mathrm{A}}^{\prime}}+m_{\mathrm{B}} \overrightarrow{v_{\mathrm{B}}^{\prime}}}_{0+0}=m_{\mathrm{A}}^{\prime} \vec{v}_{\mathrm{A}}^{\prime}+m_{\mathrm{B}} \overrightarrow{v_{\mathrm{B}}^{\prime}} \\
\vec{v}_{\mathrm{A}}^{\prime} & =-\frac{m_{\mathrm{B}}{\overrightarrow{v_{\mathrm{B}}^{\prime}}}_{m_{\mathrm{A}}^{\prime}}}{}
\end{aligned}
$$

Substitute values and solve.

$$
\begin{aligned}
& \vec{v}_{\mathrm{A}}^{\prime}=-\frac{\left(6.64 \times 10^{-27}\right)\left(1.42 \times 10^{4}\right)[\text { forward }]}{\left(3.95 \times 10^{-25} \mathrm{~kg}-6.64 \times 10^{-27} \mathrm{~kg}\right)} \\
& \vec{v}_{\mathrm{A}}^{\prime}=-242.78 \mathrm{~m} / \mathrm{s}[\text { forward }] \\
& \vec{v}_{\mathrm{A}}^{\prime} \cong-243 \mathrm{~m} / \mathrm{s}[\text { forward }]
\end{aligned}
$$

The uranium atom recoils with a velocity of $-243 \mathrm{~m} / \mathrm{s}$ after emitting the alpha particle, in the direction opposite to the emitted alpha particle.

Validate the Solution
The mass of the uranium atom is greater than the mass of the alpha particle by a factor of about $\frac{3.95 \times 10^{-25}}{\left(6.64 \times 10^{-27}\right)} \approx 60$, so it is expected that the recoil of the uranium atom will be smaller than the velocity of the alpha particle by the same factor. Dividing the velocity of the alpha particle by that of the uranium atom, $\frac{1.42 \times 10^{4}}{243}=58$, is in agreement with the expectations. (The small offset is due to the difference in the mass of the uranium atom before and after the alpha particle is emitted. Computing the mass ratio using the mass of the uranium atom after the alpha particle is emitted gives a factor of 58 , as expected.)

## Practice Problem Solutions

## Student Textbook page 322

## 30.Conceptualize the Problem

- Momentum is always conserved in a collision.
- If the collision is elastic, kinetic energy must also be conserved.
- Momentum is conserved in the $x$ and $y$ directions independently, so, in this case, the motion of the second ball after the collision will be in the same direction as the first ball before the collision.
- The total momentum of the system (ball A and B) before the collision is carried by ball $A$.
- After the collision, ball A has zero momentum.


## Identify the Goal

Is the total kinetic energy of the system before the collision, $\mathrm{E}_{\mathrm{kA}}$, equal to the total kinetic energy of the system after the collision, $\mathrm{E}_{\mathrm{kA}}^{\prime}+\mathrm{E}_{\mathrm{kB}}^{\prime}$.

## Identify the Variables

Known
$m_{\mathrm{A}}=0.155 \mathrm{~kg}$
$m_{\mathrm{B}}=0.155 \mathrm{~kg}$
$\vec{v}_{\mathrm{A}}=12.5 \mathrm{~m} / \mathrm{s}[$ in some direction]
$\vec{v}_{\mathrm{B}}=0 \mathrm{~m} / \mathrm{s}$
$\bar{v}_{\mathrm{A}}^{\prime}=0 \mathrm{~m} / \mathrm{s}$

## Develop a Strategy

Write the expression for the conservation of momentum. (Because the motion is in one direction, consider the vectors as scalars.)
Note that the initial velocity of ball B and

## Calculations

$$
m_{\mathrm{A}} v_{\mathrm{A}}+m_{\mathrm{B}} v_{\mathrm{B}}=m_{\mathrm{A}} v_{\mathrm{A}}^{\prime}+m_{\mathrm{B}} v_{\mathrm{B}}^{\prime}
$$

$$
m_{\mathrm{A}} v_{\mathrm{A}}+0=0+m_{\mathrm{B}} v_{\mathrm{B}}^{\prime}
$$

$$
v_{\mathrm{A}}=v_{\mathrm{B}}^{\prime}
$$ the final velocity of ball A are both zero.

The initial velocity of ball A before the collision is entirely transferred to ball B after the collision.
Therefore, the kinetic energy before the collision is equal to the kinetic energy after the collision and the collision is elastic.

## Validate the Solution

The solution is reasonable. This is a common occurrence in billiards: the cue ball hits another ball and transfers all its momentum to the second ball.

## 31. Conceptualize the Problem

- Momentum is always conserved in a collision.
- If the collision is elastic, kinetic energy must also be conserved.
- The total momentum of the system (Car A and Car B) before the collision is carried by cars individually, and after the collision it is carried by the combined wreckage.


## Identify the Goals

The total momentum of the system, $\vec{p}_{\mathrm{AB}}^{\prime}$, after the collision.
Whether the total kinetic energy of the system before the collision, $\mathrm{E}_{\mathrm{kA}}+\mathrm{E}_{\mathrm{kB}}$, is equal to the total kinetic energy of the system after the collision, $\mathrm{E}_{\mathrm{kA}}^{\prime}+\mathrm{E}_{\mathrm{kB}}^{\prime}$.

## Identify the Variables

Known
$\begin{array}{ll}m_{\mathrm{A}}=1735 \mathrm{~kg} & \vec{v}_{\mathrm{A}}=55.5 \mathrm{~km} / \mathrm{h}[\mathrm{N}] \\ m_{\mathrm{B}}=2540 \mathrm{~kg} & \vec{v}_{\mathrm{B}}=37.7 \mathrm{~km} / \mathrm{h}[\mathrm{N}]\end{array}$

Implied Unknown

| $\vec{v}_{\mathrm{AB}}^{\prime}$ | $\vec{p}_{\mathrm{AB}}^{\prime}$ |
| :--- | :--- |
| $E_{\mathrm{kA}}$ | $E_{\mathrm{kB}}$ |
| $E_{\mathrm{kA}}^{\prime}$ | $E_{\mathrm{kB}}^{\prime}$ |

## Develop a Strategy

Write the expression for the law of conservation of momentum. The combined momentum after the collision can be found from the momentum of the cars before the collision.

The combined momentum of the cars after the collision is $1.92 \times 105 \mathrm{~kg} \cdot \mathrm{~km} / \mathrm{h}$.
Use the law of conservation of momentum to determine the speed of the combined cars after the collision.

$$
\begin{aligned}
& v_{\mathrm{AB}}^{\prime}=\frac{m_{\mathrm{A}} v_{\mathrm{A}}+m_{\mathrm{B}} v_{\mathrm{B}}}{\left(m_{\mathrm{A}}+m_{\mathrm{B}}\right)} \\
& v_{\mathrm{AB}}^{\prime}=\frac{(1735 \mathrm{~kg})(55.5 \mathrm{~km} / \mathrm{h})+(2540 \mathrm{~kg})(37.7 \mathrm{~km} / \mathrm{h})}{(1735 \mathrm{~kg}+2540 \mathrm{~kg})} \\
& v_{\mathrm{AB}}^{\prime}=44.924 \mathrm{~km} / \mathrm{h}
\end{aligned}
$$

Calculate the combined kinetic energy of the cars before the collision.

Calculate the sum of the kinetic energies of the cars after the collision.

$$
\begin{aligned}
& E_{\mathrm{k}}=E_{\mathrm{kA}}+E_{\mathrm{kB}} \\
& E_{\mathrm{k}}=\frac{1}{2} m_{\mathrm{A}} v_{\mathrm{A}}^{2}+\frac{1}{2} m_{\mathrm{B}} v_{\mathrm{B}}^{2}
\end{aligned}
$$

$$
\begin{aligned}
& E_{\mathrm{k}}=\frac{1}{2}(1735 \mathrm{~kg})\left(55.5 \frac{\mathrm{~km}}{\mathrm{~h}}\right)^{2}+\frac{1}{2}(2540 \mathrm{~kg})\left(37.7 \frac{\mathrm{~km}}{\mathrm{~h}}\right)^{2} \\
& E_{\mathrm{k}}=4.48 \times 10^{6} \mathrm{~kg} \mathrm{~km}^{2} / \mathrm{h}^{2}
\end{aligned}
$$

$$
\begin{aligned}
& E_{\mathrm{k}}^{\prime}=\frac{1}{2}\left(m_{\mathrm{A}}+m_{\mathrm{B}}\right) v_{\mathrm{AB}}^{\prime 2} \\
& E_{\mathrm{k}}^{\prime}=\frac{1}{2}(1735+2540 \mathrm{~kg})\left(44.924 \frac{\mathrm{~km}}{\mathrm{~h}}\right)^{2} \\
& E_{\mathrm{k}}^{\prime}=4.314 \times 10^{6} \mathrm{~kg} \mathrm{~km}^{2} / \mathrm{h}^{2}
\end{aligned}
$$

Calculate the percent different between the initial total KE and final total KE.

Because the final KE of the system differs from the initial KE by almost $4 \%$, the collision is not elastic.
Validate the Solution
Because this is a collision between cars, it is expected that some kinetic energy will be lost to deforming the cars during the collision, and the collision is not expected to be elastic, so the result is reasonable.

## Chapter 7 Review

## Answers to Problems for Understanding

## Student Textbook pages 328-329

20. The mass was pulled 0.36 m from its equilibrium position before it was released. When the mass passes the equilibrium position, it is moving at maximum speed and all of its energy is kinetic. The kinetic energy was all elastic potential energy when your lab partner pulled the mass from its equilibrium position.

$$
\begin{aligned}
& E_{\mathrm{e}}=E_{\mathrm{k}} \\
& \frac{1}{2} k x^{2}=\frac{1}{2} m v^{2} \\
& x^{2}=\frac{m v^{2}}{k} \\
& x=\sqrt{\frac{(0.50 \mathrm{~kg})\left(3.375 \frac{\mathrm{~m}}{\mathrm{~s}}\right)^{2}}{45 \frac{\mathrm{~N}}{\mathrm{~m}}}} \\
& x \cong 0.36 \mathrm{~m}
\end{aligned}
$$

21. (i) Using the cosine function, $\cos \theta=\frac{\text { adj }}{\text { hyp }}$, the vertical distance down from the point of attachment is

$$
\begin{aligned}
d & =(\text { hyp })(\cos \theta) \\
& =(3.00 \mathrm{~m})\left(\cos 45^{\circ}\right) \\
& =2.12 \mathrm{~m}
\end{aligned}
$$

(ii) Thus, the vertical height up from the lowest point of its swing is $\Delta h=$ length of string - vertical distance down

$$
\begin{aligned}
& =3.00 \mathrm{~m}-2.12 \mathrm{~m} \\
& =0.88 \mathrm{~m}
\end{aligned}
$$

(iii) The gravitational potential energy of the pendulum relative to its rest position:

$$
\begin{aligned}
E_{\mathrm{g}} & =m g \Delta h \\
& =(2.00 \mathrm{~kg})\left(9.81 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}\right)(0.88 \mathrm{~m}) \\
& =17.2656 \mathrm{~J}
\end{aligned}
$$

(iv) Assuming the conservation of mechanical energy, the speed of the pendulum at the rest position is:

$$
\begin{aligned}
E_{\mathrm{T}(\mathrm{bottom})} & =E_{\mathrm{T} \text { (top) }} \\
E_{\mathrm{g}(\mathrm{~b})}+E_{\mathrm{k}(\mathrm{~b})} & =E_{\mathrm{g}(\mathrm{t})}+E_{\mathrm{k}(\mathrm{t})} \\
\left.0 \mathrm{~J}+\frac{1}{2} m v_{(\mathrm{b})}\right)^{2} & =m \mathrm{~g} \Delta h+0 \mathrm{~J} \\
v_{(\mathrm{b})} & =\sqrt{\frac{2 E_{\mathrm{k}(\mathrm{~b})}}{m}} \\
& =\sqrt{\frac{2(17.2656) \mathrm{J}}{2.00 \mathrm{~kg}}} \\
& =4.16 \frac{\mathrm{~m}}{\mathrm{~s}}
\end{aligned}
$$

22. Since there is no friction, mechanical energy is conserved. Thus,

$$
\begin{aligned}
E_{\mathrm{T}(\text { loop })} & =E_{\mathrm{T}(\text { top })} \\
E_{\mathrm{g}(\text { loop })}+E_{\mathrm{k}(\text { loop })} & =E_{\mathrm{g}(\text { top })}+E_{\mathrm{k}(\text { top })} \\
m g \Delta h_{(\mathrm{l})}+\frac{1}{2} m v_{(\mathrm{l})}^{2} & =m g \Delta h_{(\mathrm{t})}+\frac{1}{2} m v_{(\mathrm{t})^{2}}^{2} \\
\left(9.81 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}\right)(30 \mathrm{~m})+\frac{1}{2} v_{(\mathrm{l})}^{2} & =\left(9.81 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}\right)(70 \mathrm{~m})+\frac{1}{2}(0)^{2} \\
\left(294.3 \frac{\mathrm{~J}}{\mathrm{~kg}}\right)+\frac{1}{2} v_{(\mathrm{l})}^{2} & =\left(686.7 \frac{\mathrm{~J}}{\mathrm{~kg}}\right) \\
v_{(\mathrm{l})} & =\sqrt{2\left(686.7 \frac{\mathrm{~J}}{\mathrm{~kg}}-294.3 \frac{\mathrm{~J}}{\mathrm{~kg}}\right)} \\
& =28.0 \frac{\mathrm{~m}}{\mathrm{~s}} \\
& \cong 3 \times 10^{1} \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

23. The speed of the pendulum at its lowest point is $1.3 \mathrm{~m} / \mathrm{s}$.

Conservation of mechanical energy

$$
\begin{aligned}
& E_{\mathrm{k}}^{\prime}+E_{\mathrm{g}}^{\prime}=E_{\mathrm{k}}+E_{\mathrm{g}} \\
& \frac{1}{2} m v^{2}+0=0+m g h \\
& v=\sqrt{2 g h} \\
& v=\sqrt{2\left(9.81 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}\right)(0.085 \mathrm{~m})} \\
& v \cong 1.3 \frac{\mathrm{~m}}{\mathrm{~s}}
\end{aligned}
$$

24. The pen will rise at $0.78 \mathrm{~m} / \mathrm{s}$ when it is first released.

Conservation of mechanical energy

$$
\begin{aligned}
& E_{\mathrm{k}}^{\prime}+E_{\mathrm{g}}^{\prime}+E_{\mathrm{e}}^{\prime}=E_{\mathrm{k}}+E_{\mathrm{g}}+E_{\mathrm{e}} \\
& \frac{1}{2} m v^{2}+0+0=0+0+\frac{1}{2} k x^{2} \\
& v=\sqrt{\frac{k x^{2}}{m}} \\
& v=\sqrt{\frac{\left(1200 \frac{\mathrm{~N}}{\mathrm{~m}}\right)(0.005 \mathrm{~m})^{2}}{0.0500 \mathrm{~kg}}} \\
& v=0.7746 \frac{\mathrm{~m}}{\mathrm{~s}} \\
& v \cong 0.78 \frac{\mathrm{~m}}{\mathrm{~s}}
\end{aligned}
$$

The pen will rise to a maximum height of 0.031 m . At maximum height, the kinetic energy will be zero.
Conservation of mechanical energy

$$
\begin{aligned}
& E_{\mathrm{k}}^{\prime}+E_{\mathrm{g}}^{\prime}+E_{\mathrm{e}}^{\prime}=E_{\mathrm{k}}+E_{\mathrm{g}}+E_{\mathrm{e}} \\
& 0+m g \Delta h+0=0+0+\frac{1}{2} k x^{2} \\
& \Delta h=\frac{k x^{2}}{2 m g} \\
& \Delta h=\frac{\left(1200 \frac{\mathrm{~N}}{\mathrm{~m}}\right)(0.0050 \mathrm{~m})^{2}}{2(0.05 \mathrm{~kg})\left(9.81 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}\right)} \\
& \Delta h \cong 0.031 \mathrm{~m}
\end{aligned}
$$

25. Assume a horizontal position for the spring and the ball. The ball can be given a speed of $25 \mathrm{~m} / \mathrm{s}$.
Conservation of mechanical energy

$$
\begin{aligned}
& E_{\mathrm{k}}^{\prime}+E_{\mathrm{e}}^{\prime}=E_{\mathrm{k}}+E_{\mathrm{e}} \\
& \frac{1}{2} m v^{2}+0=0+\frac{1}{2} k x^{2} \\
& v^{2}=\frac{k x^{2}}{m} \\
& v=\sqrt{\frac{\left(950 \frac{\mathrm{~N}}{\mathrm{~m}}\right)(0.20 \mathrm{~m})^{2}}{1.5 \mathrm{~kg}}} \\
& v \cong 5 \frac{\mathrm{~m}}{\mathrm{~s}}
\end{aligned}
$$

26. (a) The gravitational potential energy of the skater decreased by $8.7 \times 10^{2} \mathrm{~J}$.

Select the zero point for gravitational potential energy at the final height reached.

$$
\begin{aligned}
& E_{\mathrm{k}}^{\prime}+E_{\mathrm{g}}^{\prime}=E_{\mathrm{k}}+E_{\mathrm{g}}+W_{\mathrm{ncf}} \\
& E_{\mathrm{g}}^{\prime}-E_{\mathrm{g}}=E_{\mathrm{k}}-E_{\mathrm{k}}^{\prime}+W_{\mathrm{ncf}} \\
& \Delta E_{\mathrm{g}}=\frac{1}{2} m v^{2}-\frac{1}{2} m v^{\prime 2}+W_{\mathrm{ncf}} \\
& \Delta E_{\mathrm{g}}=\frac{m}{2}\left(v^{2}-v^{\prime 2}\right)+W_{\mathrm{ncf}} \\
& \Delta E_{\mathrm{g}}=\frac{48.0 \mathrm{~kg}}{2}\left[\left(2.2 \frac{\mathrm{~m}}{\mathrm{~s}}\right)^{2}-\left(5.9 \frac{\mathrm{~m}}{\mathrm{~s}}\right)^{2}\right]-150 \mathrm{~J} \\
& \Delta E_{\mathrm{g}} \cong-8.7 \times 10^{2} \mathrm{~J}
\end{aligned}
$$

(b) The skater's initial height was +1.8 m and her final height was 0 m . Therefore, her height decreased by 1.8 m .

$$
\begin{aligned}
& \Delta E_{\mathrm{g}}=m g \Delta h^{\prime}-m g \Delta h \\
& \Delta E_{\mathrm{g}}=0-m g \Delta h \\
& \Delta h=\frac{\Delta E_{\mathrm{g}}}{-m g} \\
& \Delta h=\frac{-869.28 \mathrm{~J}}{-(48.0 \mathrm{~kg})\left(9.81 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}\right)} \\
& \Delta h=1.8461 \mathrm{~m} \\
& \Delta h \cong 1.8 \mathrm{~m}
\end{aligned}
$$

27. Velocity of the skateboard and skateboarder

$$
\begin{aligned}
m_{\mathrm{s}} \vec{v}_{\mathrm{s}}^{\prime}+m_{\mathrm{b}} \vec{v}_{\mathrm{b}}^{\prime} & =\left(m_{\mathrm{s}}+m_{\mathrm{b}}\right) \vec{v}_{\mathrm{sb}} \\
\overrightarrow{v_{\mathrm{sb}}} & =\frac{m_{\mathrm{s}}{\overrightarrow{v_{s}}}^{\prime}+m_{\mathrm{b}}{\overrightarrow{v_{\mathrm{b}}^{\prime}}}_{\left(m_{\mathrm{s}}+m_{\mathrm{b}}\right)}^{(20.0}}{\overrightarrow{v_{\mathrm{sb}}}}=\frac{(48.0 \mathrm{~kg})\left(3.2 \frac{\mathrm{~m}}{\mathrm{~s}}[\mathrm{E}]\right)+(7.0 \mathrm{~kg})\left(2.6 \frac{\mathrm{~m}}{\mathrm{~s}}[\mathrm{E}]\right)}{(48.0 \mathrm{~kg}+7.0 \mathrm{~kg})} \\
\overrightarrow{v_{\mathrm{s}}} & \cong 3.1 \frac{\mathrm{~m}}{\mathrm{~s}}[\mathrm{E}]
\end{aligned}
$$

28. Recoil velocity of the toboggan

$$
\begin{aligned}
m_{\mathrm{t}} \vec{v}_{\mathrm{t}}+m_{\mathrm{A}} \vec{v}_{\mathrm{A}} & =m_{\mathrm{t}} v_{\mathrm{t}}^{\prime}+m_{\mathrm{A}}{\overrightarrow{v_{\mathrm{A}}}}^{\prime} \\
0+0 & =m_{\mathrm{t}} \vec{v}_{\mathrm{t}}^{\prime}+m_{\mathrm{A}} \vec{v}_{\mathrm{A}}^{\prime} \\
\vec{v}_{\mathrm{t}}^{\prime} & =\frac{-\left(m_{\mathrm{A}} \vec{v}_{\mathrm{A}}^{\prime}\right)}{m_{\mathrm{t}}} \\
\vec{v}_{\mathrm{t}}^{\prime} & =\frac{-(37.0 \mathrm{~kg})\left(0.50 \frac{\mathrm{~m}}{\mathrm{~s}}[\text { forward }]\right)}{8.0 \mathrm{~kg}} \\
\vec{v}_{\mathrm{t}}^{\prime} & =2.31 \frac{\mathrm{~m}}{\mathrm{~s}}[\text { backward }]
\end{aligned}
$$

29. Velocity of the submarine immediately after it fires the torpedo

$$
\begin{aligned}
& m_{\mathrm{s}} \overrightarrow{v_{s}}+m_{\mathrm{t}} \overrightarrow{v_{\mathrm{t}}}=m_{\mathrm{s}} \vec{v}_{\mathrm{s}}^{\prime}+m_{\mathrm{t}} \vec{v}_{\mathrm{t}}^{\prime} \\
& \left(m_{s}+m_{\mathrm{t}}\right) \overrightarrow{v_{s}}=m_{\mathrm{s}}{\overrightarrow{v_{s}}}^{\prime}+m_{\mathrm{t}} \vec{v}_{\mathrm{t}}^{\prime} \\
& \vec{v}_{\mathrm{s}}^{\prime}=\frac{\left(m_{\mathrm{s}}+m_{\mathrm{t}}\right) \overrightarrow{v_{s}}-m_{\mathrm{t}} \overrightarrow{v_{t}}}{m_{\mathrm{t}}^{\prime}} \\
& \vec{v}_{\mathrm{s}}^{\prime}=\frac{\left(6.0 \times 10^{4} \mathrm{~kg}\right)\left(1.5 \frac{\mathrm{~m}}{\mathrm{~s}}\right)-\left(5.0 \times 10^{2} \mathrm{~kg}\right)\left(21 \frac{\mathrm{~m}}{\mathrm{~s}}+1.5 \frac{\mathrm{~m}}{\mathrm{~s}}\right)}{\left(6.0 \times 10^{4} \mathrm{~kg}-5.0 \times 10^{2} \mathrm{~kg}\right)} \\
& \vec{v}_{\mathrm{s}}{ }^{\prime} \cong 1.3 \frac{\mathrm{~m}}{\mathrm{~s}} \text { [forward] }
\end{aligned}
$$

30. Goalkeeper's forward velocity

$$
\begin{aligned}
m_{\mathrm{g}} \overrightarrow{v_{\mathrm{g}}}+m_{\mathrm{b}} \overrightarrow{v_{\mathrm{b}}} & =m_{\mathrm{g}} \vec{v}_{\mathrm{g}}^{\prime}+m_{\mathrm{b}} \vec{v}_{\mathrm{b}}^{\prime} \\
m_{\mathrm{g}} \overrightarrow{v_{\mathrm{g}}}+m_{\mathrm{b}} \overrightarrow{v_{\mathrm{b}}} & =0 \\
\overrightarrow{v_{\mathrm{g}}} & =\frac{-m_{\mathrm{b}} \overrightarrow{v_{\mathrm{b}}}}{m_{\mathrm{g}}} \\
\overrightarrow{v_{\mathrm{g}}} & =\frac{-(0.40 \mathrm{~kg})\left(32 \frac{\mathrm{~m}}{\mathrm{~s}}\right)}{75.0 \mathrm{~kg}} \\
\overrightarrow{v_{\mathrm{g}}} & \cong 0.17 \frac{\mathrm{~m}}{\mathrm{~s}}
\end{aligned}
$$

31. (a) Your colleague's velocity after catching the hammer is $0.21 \mathrm{~m} / \mathrm{s}$ [forward].

$$
\begin{aligned}
m_{\mathrm{c}} \overrightarrow{v_{\mathrm{c}}}+m_{\mathrm{h}} \overrightarrow{v_{\mathrm{h}}} & =\left(m_{\mathrm{c}}+m_{\mathrm{h}}\right) \vec{v}_{\mathrm{ch}}^{\prime} \\
\vec{v}_{\mathrm{ch}}^{\prime} & =\frac{m_{\mathrm{c}} \overrightarrow{v_{\mathrm{c}}}+m_{\mathrm{h}} \overrightarrow{v_{\mathrm{h}}}}{\left(m_{\mathrm{c}}+m_{\mathrm{h}}\right)} \\
\vec{v}_{\mathrm{ch}}^{\prime} & =\frac{(3.0 \mathrm{~kg})\left(4.5 \frac{\mathrm{~m}}{\mathrm{~s}}[\text { forward }]\right)}{63.0 \mathrm{~kg}} \\
\vec{v}_{\mathrm{ch}}^{\prime} & \cong 0.21 \frac{\mathrm{~m}}{\mathrm{~s}}[\text { forward }]
\end{aligned}
$$

(b) The impulse that the hammer exerted on your colleague was $13.5 \frac{\mathrm{~kg} \cdot \mathrm{~m}}{\mathrm{~s}}$.

$$
\begin{aligned}
& \vec{J}=\Delta \stackrel{\rightharpoonup}{p} \\
& \vec{J}=\left(m_{\mathrm{c}}+m_{\mathrm{h}}\right) \stackrel{\rightharpoonup}{v}_{\mathrm{ch}}^{\prime}-m_{\mathrm{c}} \vec{v}_{\mathrm{c}} \\
& \vec{J}=(63.0 \mathrm{~kg})\left(0.214 \frac{\mathrm{~m}}{\mathrm{~s}}[\text { forward }]\right)-0 \\
& \vec{J} \cong 13.5 \frac{\mathrm{~kg} \cdot \mathrm{~m}}{\mathrm{~s}}[\text { forward }]
\end{aligned}
$$

(c) $95 \%$ of the kinetic energy was lost in the collision.

$$
\begin{array}{ll}
E_{\mathrm{k}(\text { before })}=0+\frac{1}{2} m_{\mathrm{h}} v_{\mathrm{h}}^{2} & E_{\mathrm{k}(\text { affer })}=\frac{1}{2}\left(m_{\mathrm{c}}+m_{\mathrm{h}}\right) v_{\mathrm{ch}}{ }^{2} \\
E_{\mathrm{k} \text { (before) }}=\frac{1}{2}(3.0 \mathrm{~kg})\left(4.5 \frac{\mathrm{~m}}{\mathrm{~s}}\right)^{2} & E_{\mathrm{k}(\text { affer })}=\frac{1}{2}(63.0 \mathrm{~kg})\left(0.214 \frac{\mathrm{~m}}{\mathrm{~s}}\right)^{2} \\
E_{\mathrm{k}(\text { before })}=30.375 \mathrm{~J} & E_{\mathrm{k}(\text { affer })}=1.443 \mathrm{~J}
\end{array}
$$

$$
\begin{aligned}
\text { Percent of kinetic energy lost in the collision } & =\frac{30.373 \mathrm{~J}-1.443 \mathrm{~J}}{30.375 \mathrm{~J}} \times 100 \% \\
& =95.25 \%
\end{aligned}
$$

