Chapter 10

Applications of Forces

Practice Problem Solutions

Student Textbook page 459

1. (a) Frame the Problem

- Make a sketch of the vector.
- The angle is between 0° and 90° so it is in the first quadrant. Therefore, both the *x*-component and the *y*-component will be positive.
- Use trigonometric functions to find components of the vector.

Identify the Goal

The components Δd_x, Δd_y of vector $\Delta\vec{d}$

Variables and Constants

Known	Unknown
$\Delta\vec{d} = 16$ m	Δd_x
$\theta = 75°$	Δd_y

Strategy	Calculations
Draw the vector with its tail at the origin of an *x*-*y*-coordinate system.	
Draw lines from the tip of the vector to each axis, so that they are parallel to the axes.	
Calculate the components.	$\Delta d_x = \lvert\Delta\vec{d}\rvert \cos\theta$ $\Delta d_x = 16\text{ m } \cos 75°$ $\Delta d_x = 4.14$ m $\Delta d_y = \lvert\Delta\vec{d}\rvert \sin\theta$ $\Delta d_y = 16\text{ m } \sin 75°$ $\Delta d_y = 15.45$ m
Determine the signs of the components	The *x*-component lies on the positive *x*-axis so it is positive. The *y*-component lies on the positive *y*-axis so it is positive.

(a) The *x*-component of the vector is +4.1 m and the *y*-component of the vector is +15 m.

Validate

Use the Pythagorean theorem to check your answers.

$$\lvert\Delta\vec{d}\rvert^2 = \Delta d_x^2 + \Delta d_y^2$$

$$\lvert\Delta\vec{d}\rvert^2 = (4.14\text{ m})^2 + (15.45\text{ m})^2$$

$$\lvert\Delta\vec{d}\rvert^2 = 255.842$$

$$\lvert\Delta\vec{d}\rvert = 16\text{ m}$$

The value agrees with original vector.

1. (b) Frame the Problem

- Make a sketch of the vector.
- The angle is between 90° and 180° so it is in the second quadrant. Therefore, the x-component will be negative and the y-component will be positive.
- Use trigonometric functions to find components of the vector.

Identify the Goal

The components a_x, a_y of vector $\vec{a}$.

Variables and Constants

Known	Unknown
$\vec{a} = 8.1 \frac{m}{s^2}$	a_x
$\theta = 145°$	a_y

Strategy	Calculations
Draw the vector with its tail at the origin of an x-y-coordinate system.	
Identify the angle with the closest x-axis and label it θ_R	$\theta_R = 180° - 145°$ $\theta_R = 35°$
Draw lines from the tip of the vector to each axis, so that they are parallel to the axes.	
Calculate the components.	$a_x = \lvert\vec{a}\rvert \cos\theta_R$ $a_x = 8.1 \frac{m}{s^2} \cos 35°$ $a_x = 6.635 \frac{m}{s^2}$ $a_y = \lvert\vec{a}\rvert \sin\theta_R$ $a_y = 8.1 \frac{m}{s^2} \sin 35°$ $a_y = 4.646 \frac{m}{s^2}$
Determine the signs of the components	The x-component lies on the negative x-axis so it is negative. The y-component lies on the positive y-axis so it is positive.

(b) The x-component of the vector is -6.6 m/s^2 and the y-component of the vector is $+4.6$ m/s^2.

Validate

Use the Pythagorean theorem to check your answers.

$$\lvert\vec{a}\rvert^2 = a_x^2 + a_y^2$$
$$\lvert\vec{a}\rvert^2 = (6.635 \tfrac{m}{s^2})^2 + (4.646 \tfrac{m}{s^2})^2$$
$$\lvert\vec{a}\rvert^2 = 65.6085 (\tfrac{m}{s^2})^2$$
$$\lvert\vec{a}\rvert = 8.1 \tfrac{m}{s^2}$$

The value agrees with the magnitude of the original vector.

1. (c) Frame the Problem

- Make a sketch of the vector.
- The angle is between 180° and 270° so it is in the third quadrant. Therefore, the x-component will be negative and the y-component will be negative.
- Use trigonometric functions to find components of the vector.

Identify the Goal

The components v_x, v_y of vector $\vec{v}$.

Variables and Constants

Known	Unknown
$\vec{v} = 16.0\ \frac{m}{s}$	v_x
$\theta = 225°$	v_y

Strategy	Calculations
Draw the vector with its tail at the origin of an *x*-*y*-coordinate system.	
Identify the angle with the closest *x*-axis and label it θ_R	$\theta_R = 225° - 180°$ $\theta_R = 45°$
Draw lines from the tip of the vector to each axis, so that they are parallel to the axes.	
Calculate the components.	$v_x = \lvert\vec{v}\rvert \cos\theta_R$ $v_x = 16.0\ \frac{m}{s} \cos 45°$ $v_x = 11.314\ \frac{m}{s}$ $v_y = \lvert\vec{v}\rvert \sin\theta_R$ $v_y = 16.0\ \frac{m}{s} \sin 45°$ $v_y = 11.314\ \frac{m}{s}$
Determine the signs of the components	The *x*-component lies on the negative *x*-axis so it is negative. The *y*-component lies on the negative *y*-axis so it is negative.

(c) The *x*-component of the vector is −11.3 m/s and the *y*-component of the vector is −11.3 m/s.

Validate

Use the Pythagorean theorem to check your answers.

$$\lvert\vec{v}\rvert^2 = v_x^2 + v_y^2$$
$$\lvert\vec{v}\rvert^2 = (11.314\ \tfrac{m}{s})^2 + (11.314\ \tfrac{m}{s})^2$$
$$\lvert\vec{v}\rvert^2 = 256.013(\tfrac{m}{s})^2$$
$$\lvert\vec{v}\rvert = 16.0\ \tfrac{m}{s}$$

The value agrees with the magnitude of the original vector.

2. (a) Frame the Problem

- Make a sketch of the vector.
- The vector is described in compass directions.
- Convert to an *x*-*y*-coordinate system. Let $+y$ be north, and $-y$ south. East will become $+x$ and west $-x$.
- You will need to find the angle with the closest *x*-axis.

Identify the Goal

The components Δd_x, Δd_y of vector $\Delta\vec{d} = 20.0$ km[N20.0°E].

Variables and Constants

Known	Unknown
$\Delta\vec{d} = 20.0$ km[N20.0°E]	Δd_x
	Δd_y
	θ

Strategy	**Calculations**
Draw an *x*-*y*-coordinate system and indicate that the axes also represent compass directions.	
Draw the vector with its tail at the origin.	
Identify the angle, θ.	$\theta = 90° - 20.0°$ $\theta = 70°$
Use the 20.0° made by the vector to the *y*-axis to find the angle the vector makes with the *x*-axis.	
Draw lines from the tip of the vector to each axis, so that they are parallel to the axes.	
Calculate the components.	$\Delta d_x = \lvert\Delta\vec{d}\rvert \cos\theta$ $\Delta d_x = 20.0\ \text{km} \cos 70°$ $\Delta d_x = 6.84\ \text{km}$ $\Delta d_y = \lvert\Delta\vec{d}\rvert \sin\theta$ $\Delta d_y = 20.0\ \text{km}\ \sin 70°$ $\Delta d_y = 18.794\ \text{km}$
Determine the signs of the components	The *x*-component lies on the positive *x*-axis so it is positive. The *y*-component lies on the positive *y*-axis so it is positive.

(a) The *x*-component of the vector is +6.84 km and the *y*-component of the vector is +18.8 km.

Validate

Use the Pythagorean theorem to check your answers.

$$\lvert\Delta\vec{d}\rvert^2 = \Delta d_x^2 + \Delta d_y^2$$
$$\lvert\Delta\vec{d}\rvert^2 = (6.840\ \text{km})^2 + (18.794\ \text{km})^2$$
$$\lvert\Delta\vec{d}\rvert^2 = 400.000$$
$$\lvert\Delta\vec{d}\rvert = 20.0\ \text{km}$$

The value agrees with original vector.

2. (b) Frame the Problem

- Make a sketch of the vector.
- The vector is described in compass directions.
- Convert to an *x*-*y*-coordinate system. Let $+y$ be north, and $-y$ south. East will become $+x$ and west will be $-x$.
- You will need to find the angle with the closest *x*-axis.

Identify the Goal

The components v_x, v_y of vector $\vec{v} = 3.0$ m/s[E30.0°S]

Variables and Constants

Known	Unknown
$\vec{v} = 16.0\ \frac{\text{m}}{\text{s}}$[E30.0°S]	v_x
	v_y
	θ

Strategy

Draw an x-y-coordinate system and indicate that the axes also represent compass directions. Draw the vector with its tail at the origin. Identify the angle, θ.

The vector makes an angle $\theta = 30.0°$ with the x-axis.

Draw lines from the tip of the vector to each axis, so that they are parallel to the axes.

Calculate the components.

Calculations

$v_x = |\vec{v}| \cos\theta$

$v_x = 3.0\ \frac{m}{s} \cos 30°$

$v_x = 2.598\ \frac{m}{s}$

$v_y = |\vec{v}| \sin\theta$

$v_y = 3.0\ \frac{m}{s} \sin 30°$

$v_y = 1.50\ \frac{m}{s}$

Determine the signs of the components

The x-component lies on the positive x-axis so it is positive. The y-component lies on the negative y-axis so it is negative.

(b) The x-component of the vector is −2.6 m/s and the y-component of the vector is −1.5 m/s.

Validate

Use the Pythagorean theorem to check your answers.

$|\vec{v}|^2 = v_x^2 + v_y^2$

$|\vec{v}|^2 = (2.598\ \frac{m}{s})^2 + (1.50\ \frac{m}{s})^2$

$|\vec{v}|^2 = 8.9996(\frac{m}{s})^2$

$|\vec{v}| = 3.0\ \frac{m}{s}$

The value agrees with the magnitude of the original vector.

2. (c) Frame the Problem

- Make a sketch of the vector.
- The vector is described in compass directions.
- Convert to an x-y-coordinate system. Let $+y$ be north, and $-y$ south. East will become $+x$ and west will be $-x$.
- You will need to find the angle with the closest x-axis.

Identify the Goal

The components v_x, v_y of vector $\vec{v} = 6.8$ m/s[W70.0°N]

Variables and Constants

Known	Unknown
$\vec{v} = 6.8\ \frac{m}{s}$[W70.0°N]	v_x
	v_y
	θ

Strategy	Calculations
Draw an *x*-*y*-coordinate system and indicate that the axes also represent compass directions. Draw the vector with its tail at the origin. Identify the angle, θ.	
The vector makes an angle $\theta = 70.0°$ with the *x*-axis.	
Draw lines from the tip of the vector to each axis, so that they are parallel to the axes.	
Calculate the components.	$v_x = \lvert\vec{v}\rvert \cos\theta$ $v_x = 6.8\ \frac{m}{s} \cos 70°$ $v_x = 2.326\ \frac{m}{s}$ $v_y = \lvert\vec{v}\rvert \sin\theta$ $v_y = 6.8\ \frac{m}{s} \sin 70°$ $v_y = 6.390\ \frac{m}{s}$
Determine the signs of the components	The *x*-component lies on the negative *x*-axis so it is negative. The *y*-component lies on the positive *y*-axis so it is positive.

(c) The *x*-component of the vector is −2.3 m/s and the *y*-component of the vector is +6.4 m/s.

Validate

Use the Pythagorean theorem to check your answers.

$$\lvert\vec{v}\rvert^2 = v_x^2 + v_y^2$$
$$\lvert\vec{v}\rvert^2 = (2.326\ \tfrac{m}{s})^2 + (6.390\ \tfrac{m}{s})^2$$
$$\lvert\vec{v}\rvert^2 = 46.242(\tfrac{m}{s})^2$$
$$\lvert\vec{v}\rvert = 6.8\ \tfrac{m}{s}$$

The value agrees with the magnitude of the original vector.

3. Frame the Problem

- Make a sketch of the situation.
- The position of the balloon is described in compass directions.
- Resolving the vector into components will allow you to determine how the pickup vehicle should reach the balloon.
- Convert to an *x*-*y*-coordinate system. Let $+y$ be north, and $-y$ south. East will become $+x$ and west will be $-x$.
- You will need to find the angle with the closest *x*-axis.

Identify the Goal

The components Δd_x, Δd_y of vector $\Delta\vec{d} = 60.0$ km[E60.0°N].

Variables and Constants

Known	Unknown
$\Delta\vec{d} = 60.0$ km[E60.0°N]	Δd_x
	Δd_y
	θ

Strategy	**Calculations**
Draw an x-y-coordinate system and indicate that the axes also represent compass directions. Draw the vector with its tail at the origin. Identify the angle, θ.	
The vector makes an angle of $\theta = 60.0°$ with the positive x-axis.	
Draw lines from the tip of the vector to each axis, so that they are parallel to the axes.	
Calculate the components.	$\Delta d_x = \lvert\Delta\vec{d}\rvert \cos\theta$ $\Delta d_x = 60.0\ \text{km}\ \cos 60°$ $\Delta d_x = 30.0\ \text{km}$ $\Delta d_y = \lvert\Delta\vec{d}\rvert \sin\theta$ $\Delta d_y = 60.0\ \text{km}\ \sin 60°$ $\Delta d_y = 51.962\ \text{km}$
Determine the signs of the components	The x-component lies on the positive x-axis so it is positive. The y-component lies on the positive y-axis so it is positive.

The pickup vehicle will have to drive 3.0×10^1 km east and then 52 km north to pick up the balloon.

Validate

Use the Pythagorean theorem to check your answers.

$$\lvert\Delta\vec{d}\rvert^2 = \Delta d_x^2 + \Delta d_y^2$$
$$\lvert\Delta\vec{d}\rvert^2 = (30.0\ \text{km})^2 + (51.962\ \text{km})^2$$
$$\lvert\Delta\vec{d}\rvert^2 = 3600.05$$
$$\lvert\Delta\vec{d}\rvert = 60.0\ \text{km}$$

Practice Problem Solutions

Student Textbook page 463

4. **Frame the Problem**
   - Make a diagram of the problem.
   - The hiker's trip is in three segments which are given in terms of displacements and compass directions. You can convert the compass directions to the x-y-coordinate system.
   - The total displacement is the vector sum of the three displacement vectors.
   - The components of each displacement can be determined and summed to get the components of the resultant vector.

   Identify the Goal

   (a) The displacement of the boat, $\Delta\vec{d}_{\text{total}}$

   (b) The direction, θ, the boat would need in order to head straight home

Variables and Constants

Known	Unknown
$\Delta\vec{d}_A = 2.7$ km[S]	$\Delta\vec{d}_{total}$
$\Delta\vec{d}_B = 3.4$ km[S26°E]	Δd_{Ax} Δd_{Ay}
$\Delta\vec{d}_C = 1.9$ km[E12°N]	Δd_{Bx} Δd_{By}
	Δd_{Cx} Δd_{Cy}
	θ θ_A θ_B θ_C

Strategy	Calculations
Draw the first displacement on an x-y-coordinate system with $+y$ corresponding to north.	
Find the angle the vector makes with the x-axis.	Because displacement A lies along the negative y-axis, the angle $\theta_A = 0°$.
Calculate the x- and y-components of displacement A and determine the signs of the components.	Thus, the x-component will be zero, and y-component will be negative.
Draw displacement B and calculate its x- and y-components.	$\Delta d_{Ax} = 0$ $\Delta d_{Ay} = -2.7$ km
Determine the angle the vector makes with the x-axis, θ_B.	$\theta_B = 90° - 26°$ $\theta_B = 64°$
Calculate the x- and y-components of displacement B.	$\Delta d_{Bx} = \lvert\Delta\vec{d}_B\rvert \cos\theta$ $\Delta d_{Bx} = 3.4 \text{ km} \cos 64°$ $\Delta d_{Bx} = 1.490$ km $\Delta d_{By} = \lvert\Delta\vec{d}_B\rvert \sin\theta$ $\Delta d_{By} = 3.4 \text{ km} \sin 64°$ $\Delta d_{By} = 3.056$ km
Determine the signs of the components.	The vector lies in the 4th quadrant, so the x-component is positive and the y-component is negative.
Draw displacement C and calculate its x- and y-components.	
Determine the angle the vector makes with the x-axis, θ_C.	$\theta_C = 12°$
Calculate the x- and y-components of displacement B.	$\Delta d_{Cx} = \lvert\Delta\vec{d}_C\rvert \cos\theta$ $\Delta d_{Cx} = 1.9 \text{ km} \cos 12°$ $\Delta d_{Cx} = 1.858$ km $\Delta d_{Cy} = \lvert\Delta\vec{d}_C\rvert \sin\theta$ $\Delta d_{Cy} = 1.9 \text{ km} \sin 12°$ $\Delta d_{Cy} = 0.395$ km
Determine the signs of the components.	The vector lies in the first quadrant, so the x- and y-components are positive.
The components of the resultant vector can be summed individually.	$\Delta d_{total\,x} = \Delta d_{Ax} + \Delta d_{Bx} + \Delta d_{Cx}$ $\Delta d_{total\,x} = 0 + 1.490 \text{ km} + 1.858 \text{ km}$ $\Delta d_{total\,x} = 3.348$ km $\Delta d_{total\,y} = \Delta d_{Ay} + \Delta d_{By} + \Delta d_{Cy}$ $\Delta d_{total\,y} = -2.7 - 3.056 \text{ km} + 0.395 \text{ km}$ $\Delta d_{total\,y} = -5.361$ km

Use the Pythagorean theorem to determine the magnitude of the resultant displacement.

$$(\Delta d_{\text{total}})^2 = (\Delta d_{\text{total x}})^2 + (\Delta d_{\text{total y}})^2$$
$$(\Delta d_{\text{total}})^2 = (3.348 \text{ km})^2 + (-5.361 \text{ km})^2$$
$$(\Delta d_{\text{total}})^2 = 39.949 \text{ km}^2$$
$$\Delta d_{\text{total}} = 6.320 \text{ km}$$

Use the components to determine the direction of the resultant displacement.

$$\tan\theta = \frac{-5.361 \cancel{\text{km}}}{3.348 \cancel{\text{km}}}$$
$$\tan\theta = -1.6012$$
$$\theta = \tan^{-1} 1.6012$$
$$\theta = -58.01°$$

(a) The displacement is 6.3 km[E58°S].

(b) To head straight back, the boat should head in the direction [W58°N]

Validate

In each case, the units cancelled to give the correct units for the desired quantity. The displacement, 6.3 km, is less than the total distance travelled (2.7 km + 3.4 km + 1.9 km), as it should be. The solution can be checked using a graphical method.

5. Frame the Problem

- Make a diagram of the problem.
- The jet-ski wants to travel to an island and must aim further upriver to compensate for the current which is flowing to the east.
- The velocity of the jet-ski relative to the shore, $\vec{v}_{\text{js}}$, is the vector sum of the velocity of the river relative to the shore, $\vec{v}_{\text{rs}}$, and the velocity of the jet-ski relative to the river, $\vec{v}_{\text{jr}}$. The magnitude of the velocity of the jet-ski relative to the river, $\vec{v}_{\text{jr}}$, is unknown, but its direction is known.
- In other words, the resultant velocity is the sum of the jet ski's heading, and the river velocity. The magnitude of the jet-ski's heading is given, but not its direction. Conversely, the direction of the resultant velocity is given, but not its magnitude.
- The sine law can be used.

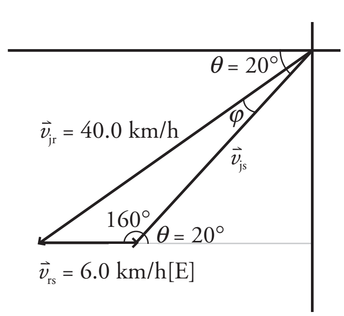
Identify the Goal

(a) The direction the jet-ski must head in order to travel to the island.

(b) The time it will take him to reach the island, Δt.

Variables and Constants
Identify the Variables

Known	Implied	Unknown
$\vec{v}_{\text{jr}} = 40.0 \text{ km/h}[\theta]$	$\theta + \varphi = 20°$	θ
$\vec{v}_{\text{rs}} = 6.0 \text{ km/h}[\text{E}]$		φ
$\Delta\vec{d} = 5.0 \text{ km/h}[\text{W}20.0°\text{S}]$		$\vec{v}_{\text{js}}$
		Δt



Develop a Strategy	Calculations
Determine how the vectors add together.	$\vec{v}_{js} = \vec{v}_{jr} + \vec{v}_{rs}$
Note that it would be difficult to use components here because we know the magnitude of $\vec{v}_{jr}$ but not its direction (θ) and we know the direction of $\vec{v}_{js}$ but not its magnitude.	
From the figure, it is evident that the sine law can be used to solve for the unknown velocity, $\vec{v}_{js}$, and the unknown direction, θ.	$\frac{\sin\varphi}{6\text{ km/h}} = \frac{\sin 160°}{40\text{ km/h}}$ $\sin\varphi = \frac{6\text{ km/h}}{40\text{ km/h}}\sin 160°$ $\sin\varphi = 0.05130$ $\varphi = \sin^{-1}(0.05130)$ $\varphi = 2.94°$
Note that $\theta + \varphi = 20°$ Solve for θ.	$\theta = 20° - \varphi$ $\theta = 20° - 2.94°$ $\theta = 17.06°$ $\theta \cong 17.1°$
(a) He should head the jet-ski in the direction [W17.1° S].	
Apply the sine law again to find the unknown velocity, $\vec{v}_{js}$	$\frac{\sin\theta}{\lvert\vec{v}_{js}\rvert} = \frac{\sin 160°}{40\text{ km/h}}$ $\lvert\vec{v}_{js}\rvert = (40\text{ km/h})\frac{\sin\theta}{\sin 160°}$ $\lvert\vec{v}_{js}\rvert = (40\text{ km/h})\frac{\sin 17.06°}{\sin 160°}$ $\lvert\vec{v}_{js}\rvert = 34.31\text{ km/h}$
Find the time using the definition for the average velocity.	$\vec{v}_{ave} = \frac{\Delta\vec{d}}{\Delta t}$ $\Delta t = \frac{\Delta\vec{d}}{\vec{v}_{ave}}$ $\Delta t = \frac{5.0\text{ km[W20.0°S]}}{34.31\text{ km/h[W20.0°S]}}$ $\Delta t = 0.1457\text{ h}$ $\Delta t = 0.1457\text{ h} \times \frac{60.0\text{ min}}{1\text{ h}}$ $\Delta t = 8.74\text{ min}$ $\Delta t \cong 8.7\text{ min}$
(b) It will take him 8.7 min to reach the island.	

Validate the Solution

Once the direction of the heading (i.e. the velocity of the jet-ski relative to the water) is found, vector addition can be used to determine the velocity of the jet-ski relative to the shore (or the resultant vector). The solution can be found from components:

x: $v_{jsx} = v_{jrx} + v_{rsx}$

or, $v_{js}\cos 20.0° = v_{jr}\cos\theta + v_{rsx}$ (1)

y: $v_{jsy} = v_{jry}$

or, $v_{js}\sin 20.0° = v_{jr}\sin\theta$ (2)

Either equation, (1) or (2) can be solved:

From equation (1):

$$v_{js}\cos 20.0° = v_{jr}\cos\theta + v_{rsx}$$
$$v_{js} = \frac{v_{jr}\cos\theta + v_{rsx}}{\cos 20.0°} = \frac{(40\text{ km/h})\cos 17.06 + (-6.0\text{ km/h})}{\cos 20.0°}$$
$$v_{js} = 34.31\text{ km/h}$$

or, from equation (2):

$$v_{js}\sin 20.0° = v_{jr}\sin\theta$$
$$v_{js} = \frac{v_{jr}\sin\theta}{\sin 20.0°} = \frac{(40.0\text{ km/h})\sin 17.06°}{\sin 20.0°}$$
$$v_{js} = 34.31\text{ km/h}$$

The magnitude of the velocity calculated here agrees with that calculated from the sine law above.

6. Frame the Problem

- Make a sketch of the motion in the problem.
- Vector addition will be used.
- The vector sum of the velocity of the cable relative to the shuttle and the velocity of the shuttle relative to the space station must be equal to the velocity of the cable relative to the space station.

Identify the Goal

The velocity of the cable relative to the space station.

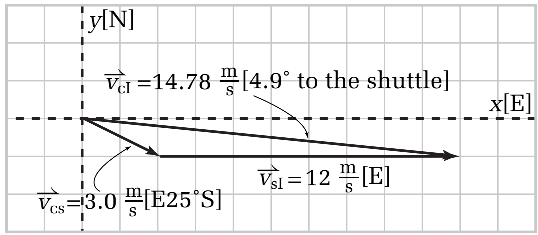
Variables and Constants

Known	Implied	Unknown
$\vec{v}_{sI} = 12$		θ
$\vec{v}_{cs} = 3.0\ \frac{\text{m}}{\text{s}}$ [25° to shuttle]		$v_{cs\,x}$
		$v_{cs\,y}$
		$v_{sI\,x}$ $v_{sI\,y}$
		$\vec{v}_{cI}$

Strategy and Calculations

Draw the vectors on a *x*-*y*-coordinate system (+*y* coincides with north).

Scale 1.0 cm : 3 m/s



Find the *x*-and *y*-components of the vectors $\vec{v}_{cs}$ and $\vec{v}_{sI}$.

$\vec{v}_{sI} = 12\ \frac{m}{s}[E]$

$\vec{v}_{cs} = 3.0\ \frac{m}{s}[E25°S]$

$\vec{v}_{cI} = \vec{v}_{cs} + \vec{v}_{sI}$

$\vec{v}_{cs} = 3.0\ \frac{m}{s}[E25°S]$

$\theta_{cs} = 25°$

$v_{cs_x} = |\vec{v}_{cs}| \cos\theta_{cs}$

$v_{cs_x} = 3.0\ \frac{m}{s} \cos 25°$

$v_{cs_x} = 3.0\ \frac{m}{s}(0.9063)$

$v_{cs_x} = 2.72\ \frac{m}{s}$

$v_{cs_y} = |\vec{v}_{cs}| \sin\theta_{cs}$

$v_{cs_y} = 3.0\ \frac{m}{s} \sin 25°$

$v_{cs_y} = 3.0\ \frac{m}{s} \sin(0.4226)$

$v_{cs_y} = 1.27\ \frac{m}{s}$

Add the components of $\vec{v}_{cs}$ and $\vec{v}_{sI}$ to obtain the components of $\vec{v}_{cI}$.

Vector	***x*-component (m/s)**	***y*-component (m/s)**
$\vec{v}_{sI}$	12.0	0.0
$\vec{v}_{cs}$	2.72	−1.27
$\vec{v}_{cI}$	14.72	−1.27

Use the Pythagorean theorem to find the magnitude of $\vec{v}_{cI}$, and find the angle the resultant makes with the *x*-axis.

$|\vec{v}_{cI}|^2 = (\vec{v}_{cI_x})^2 + (\vec{v}_{cI_y})^2$

$|\vec{v}_{cI}|^2 = \left(14.72\ \frac{m}{s}\right)^2 + \left(-1.27\ \frac{m}{s}\right)^2$

$|\vec{v}_{cI}|^2 = 216.7\ \frac{m^2}{s^2} + 1.61\ \frac{m^2}{s^2}$

$|\vec{v}_{cI}|^2 = 218.3\ \frac{m^2}{s^2}$

$|\vec{v}_{cI}| = 14.78\ \frac{m}{s}$

$|\vec{v}_{cI}| = 15\ \frac{m}{s}$

$\tan\theta = \frac{-1.27\ \frac{m}{s}}{14.72\ \frac{m}{s}}$

$\tan\theta = 0.0863$

$\theta = \tan^{-1} 0.0863$

$\theta = 4.9°$

Since the x-component is positive and the y-component is negative the vector and the angle lie in the fourth quadrant.
The velocity of the cable relative to the space station is 15 m/s in a direction 4.9° to the shuttle.

Validate
The velocity of the cable relative to the space station is correct since it should be greater than the velocity of the shuttle relative to the space station alone. The cable possesses the shuttle's velocity as well as its own.

Practice Problem Solutions

Student Textbook page 467

7. (a) Frame the Problem
- Make a diagram of the problem; use a coordinate system with East to the right and north at the top.
- Because the vector sum of the three forces is zero, the forces in each dimension must add to zero.
- Find the components of the third force that will make the sum of the x and y-components of all the forces equal to zero.

Identify the Goal
The force, $\vec{F}_3$, which will make the net force equal to zero.

Variables and Constants
Identify the Variables

Known	Unknown
$\vec{F}_1 = 154\text{ N[E22°S]}$	$\vec{F}_3$
$\vec{F}_2 = 203\text{ N[W22°N]}$	

Develop a Strategy	Calculations	
Draw each of the known force vectors with its tail on the origin of the coordinate system and determine the angle each vector makes with the nearest x axis.		
Find the x-component of each of the force vectors.	$F_{1x} = \lvert\vec{F}_1\rvert\cos 22.0°$ $F_{1x} = (154\text{ N})(0.92718)$ $F_{1x} = 142.786\text{ N}$ The angle is in the 4th quadrant so the x-component is positive.	$F_{2x} = -\lvert\vec{F}_2\rvert\cos 74.0°$ $F_{2x} = -(203\text{ N})(0.27564)$ $F_{2x} = -55.954\text{ N}$ The angle is in the 2nd quadrant so the x-component is negative.

Find the y-component of each of the force vectors.

$$F_{1y} = -|\vec{F}_1|\sin 22.0°$$
$$F_{1y} = -(154\ \text{N})(0.37461)$$
$$F_{1y} = -57.689\ \text{N}$$

The angle is in the 4th quadrant so the y-component is negative.

$$F_{2y} = |\vec{F}_2|\sin 74.0°$$
$$F_{2y} = (203\ \text{N})(0.96126)$$
$$F_{2y} = 195.136\ \text{N}$$

The angle is in the 2nd quadrant so the y-component is positive.

Sum the x and y-components individuallyto find the components of the unknown vector.

$$F_{1x} + F_{2x} + F_{3x} = 0.0$$
$$F_{3x} = 0.0 - F_{1x} - F_{2x}$$
$$F_{3x} = 0.0 - (142.786\ \text{N}) - (-55.954\ \text{N})$$
$$F_{3x} = -86.832\ \text{N}$$
$$F_{1y} + F_{2y} + F_{3y} = 0.0$$
$$F_{3y} = 0.0 - F_{1y} - F_{2y}$$
$$F_{3y} = 0.0 - (-57.689\ \text{N}) - (195.136\ \text{N})$$
$$F_{3y} = -137.447\ \text{N}$$

Use the Pythagorean Theorem to find the magnitude of $\vec{F}_3$.

$$|\vec{F}_3|^2 = (F_{3x})^2 + (F_{3y})^2$$
$$|\vec{F}_3|^2 = (-86.832\ \text{N})^2 + (-137.447\ \text{N})^2$$
$$|\vec{F}_3|^2 = 26431.474\ \text{N}^2$$
$$|\vec{F}_3| = 162.578\ \text{N}$$
$$|\vec{F}_3| \cong 163\ \text{N}$$

Use the tangent function and the magnitudes of the x- and y-components to find the angle θ_3.

$$\tan\theta = \frac{F_{3y}}{F_{3x}}$$
$$\tan\theta = \frac{-137.447\ \text{N}}{-86.832\ \text{N}}$$
$$\tan\theta = 1.5829$$
$$\theta = \tan^{-1}(1.5829)$$
$$\theta = 57.72°$$
$$\theta \cong 58°$$

The third vector is 163 N[W58°S].

Validate the Solution
From a diagram, it is evident that the third vector will be in either the first or third quadrant, so the answer is reasonable.

7. (b) Frame the Problem
- Same as above.

Identify the Goal
The force, $\vec{F}_3$, which will make the net force equal to zero.

Variables and Constants
Identify the Variables

Known

$\vec{F}_1 = 782\ \text{N[E12°N]}$

$\vec{F}_2 = 629\ \text{N[W24°S]}$

Unknown

$\vec{F}_3$

Develop a Strategy

Calculations

Draw each of the known force vectors with its tail on the origin of the coordinate system and determine the angle each vector makes with the nearest *x* axis.

Find the *x*-component of each of the force vectors.

$$F_{1x} = \left|\vec{F}_1\right| \cos 12.0° \qquad F_{2x} = -\left|\vec{F}_2\right| \cos 24.0°$$

$$F_{1x} = (782\ \text{N})(0.97815) \qquad F_{2x} = -(629\ \text{N})(0.91354$$

$$F_{1x} = 764.91\ \text{N} \qquad F_{2x} = -574.62\ \text{N}$$

The angle is in the 1st quadrant so the *x*-component is positive.

The angle is in the 3rd quadrant so the *x*-component is negative.

Find the *y*-component of each of the force vectors.

$$F_{1y} = \left|\vec{F}_1\right| \sin 12.0° \qquad F_{2y} = -\left|\vec{F}_2\right| \sin 24.0°$$

$$F_{1y} = (782\ \text{N})(0.20791) \qquad F_{2y} = -(629\ \text{N})(0.40674)$$

$$F_{1y} = 162.59\ \text{N} \qquad F_{2y} = -255.84\ \text{N}$$

The angle is in the 1st quadrant so the *y*-component is positive.

The angle is in the 3rd quadrant so the *y*-component is positive.

Sum the *x* and *y*-components individually to find the components of the unknown vector.

$$F_{1x} + F_{2x} + F_{3x} = 0.0$$

$$F_{3x} = 0.0 - F_{1x} - F_{2x}$$

$$F_{3x} = 0.0 - (764.91\ \text{N}) - (-574.62\ \text{N})$$

$$F_{3x} = -190.29\ \text{N}$$

$$F_{1y} + F_{2y} + F_{3y} = 0.0$$

$$F_{3y} = 0.0 - F_{1y} - F_{2y}$$

$$F_{3y} = 0.0 - (162.59\ \text{N}) - (-255.84\ \text{N})$$

$$F_{3y} = 93.25\ \text{N}$$

Use the Pythagorean Theorem to find the magnitude of $\vec{F}_3$.

$$\left|\vec{F}_3\right|^2 = (F_{3x})^2 + (F_{3y})^2$$

$$\left|\vec{F}_3\right|^2 = (-190.29\ \text{N})^2 + (93.25\ \text{N})^2$$

$$\left|\vec{F}_3\right|^2 = 44905.8\ \text{N}^2$$

$$\left|\vec{F}_3\right| = 211.91\ \text{N}$$

$$\left|\vec{F}_3\right| \cong 212\ \text{N}$$

Use the tangent function and the magnitudes of the x- and y-components to find the angle θ_3.

$$\tan\theta = \frac{F_{3y}}{F_{3x}}$$
$$\tan\theta = \frac{-137.447\text{ N}}{-86.832\text{ N}}$$
$$\tan\theta = 1.5829$$
$$\theta = \tan^{-1}(1.5829)$$
$$\theta = 57.72°$$
$$\theta \cong 58°$$

The third vector is 212 N[W26°N].

Validate the Solution
From a diagram, it is evident that the third vector will be in either the second or fourth quadrant, so the answer is reasonable.

7. (c) Frame the Problem
- Same as above.

Identify the Goal
The force, $\vec{F}_3$, which will make the net force equal to zero.

Variables and Constants
Identify the Variables

Known	Unknown
$\vec{F}_1 = 48\text{ N[W81°N]}$	$\vec{F}_4$
$\vec{F}_2 = 61\text{ N[E63°N]}$	
$\vec{F}_3 = 78\text{ N[E15°S]}$	

Develop a Strategy

Draw each of the known force vectors with its tail on the origin of the coordinate system and determine the angle each vector makes with the nearest x axis.

Find the x-component of each of the force vectors.

Calculations

$$F_{1x} = -\left|\vec{F}_1\right|\cos 81.0°$$
$$F_{1x} = -(48\text{ N})(0.15643)$$
$$F_{1x} = -7.5088\text{ N}$$

The angle is in the 2nd quadrant so the x-component is negative.

$$F_{2x} = \left|\vec{F}_2\right|\cos 63.0°$$
$$F_{2x} = (61\text{ N})(0.4540)$$
$$F_{2x} = 27.693\text{ N}$$

The angle is in the 1st quadrant so the x-component is positive.

$$F_{3x} = \left|\vec{F}_3\right|\cos 15.0°$$
$$F_{3x} = (78\text{ N})(0.96592)$$
$$F_{3x} = 75.342\text{ N}$$

The angle is in the 4th quadrant so the x-component is positive.

Find the y-component of each of the force vectors.

$$F_{1y} = |\vec{F}_1|\sin 81.0°$$
$$F_{1y} = (48\text{ N})(0.9877)$$
$$F_{1y} = 47.409\text{ N}$$

The angle is in the 2nd quadrant so the y-component is positive.

$$F_{2y} = |\vec{F}_2|\sin 63.0°$$
$$F_{2y} = (61\text{ N})(0.8910)$$
$$F_{2y} = 54.351\text{ N}$$

The angle is in the 1st quadrant so the y-component is positive.

$$F_{3y} = -|\vec{F}_3|\sin 15.0°$$
$$F_{3y} = -(78\text{ N})(0.25882)$$
$$F_{3y} = -20.188\text{ N}$$

The angle is in the 4th quadrant so the y-component is negative.

Sum the x and y-components individually to find the components of the unknown vector.

$$F_{1x} + F_{2x} + F_{3x} + F_{4x} = 0.0$$
$$F_{4x} = 0.0 - F_{1x} - F_{2x} - F_{3x}$$
$$F_{4x} = 0.0 - (-7.5088\text{ N}) - (27.6934\text{ N}) - (75.3422\text{ N})$$
$$F_{4x} = -95.527\text{ N}$$
$$F_{1y} + F_{2y} + F_{3y} + F_{4y} = 0.0$$
$$F_{4y} = 0.0 - F_{1y} - F_{2y} - F_{3y}$$
$$F_{4y} = 0.0 - (47.4090\text{ N}) - (54.3514\text{ N}) - (-20.1879\text{ N})$$
$$F_{4y} = -81.5725\text{ N}$$

Use the Pythagorean Theorem to find the magnitude of $\vec{F}_4$.

$$|\vec{F}_4|^2 = (F_{4x})^2 + (F_{4y})^2$$
$$|\vec{F}_4|^2 = (-95.527\text{ N})^2 + (81.5725\text{ N})^2$$
$$|\vec{F}_4|^2 = 15779.4\text{ N}^2$$
$$|\vec{F}_4| = 125.62\text{ N}$$
$$|\vec{F}_4| \cong 126\text{ N}$$

Use the tangent function and the magnitudes of the x- and y-components to find the angle θ_3.

$$\tan\theta = \frac{F_{4y}}{F_{4x}}$$
$$\tan\theta = \frac{-81.5725\text{ N}}{-95.5268\text{ N}}$$
$$\tan\theta = 0.8539$$
$$\theta = \tan^{-1}(0.8539)$$
$$\theta = 40.49°$$
$$\theta \cong 40°$$

The fourth vector is 126 N[W40°S].

Validate the Solution

From a diagram, it is evident that the fourth vector will be in the third quadrant to balance the other force vectors, so the answer is reasonable.

8. Frame the Problem

- Caitlin acts as the equilibrating force, so the question is similar to the above.

Identify the Goal

The force, $\vec{F}_C$, which will make the net force equal to zero.

Variables and Constants

Identify the Variables

Known	Unknown
$\vec{F}_A = 15\text{ N[N58°E]}$	$\vec{F}_C$
$\vec{F}_B = 18\text{ N[S23°E]}$	

Develop a Strategy

Draw each of the known force vectors with its tail on the origin of the coordinate system and determine the angle each vector makes with the nearest *x* axis.

Calculations

Amy's force is 32° to the positive *x*-axis.
Buffy's force is 67° to the positive *x*-axis.

Find the *x*-component of each of the force vectors.

$$F_{Ax} = |\vec{F}_A|\cos 32°$$
$$F_{Ax} = (15\text{ N})(0.8480)$$
$$F_{Ax} = 12.7207\text{ N}$$

The angle is in the 1st quadrant so the *x*-component is positive.

$$F_{Bx} = |\vec{F}_B|\cos 67°$$
$$F_{Bx} = (18\text{ N})(0.3907)$$
$$F_{Bx} = 7.0332\text{ N}$$

The angle is in the 4th quadrant so the *x*-component is positive.

Find the *y*-component of each of the force vectors.

$$F_{1y} = |\vec{F}_1|\sin 32°$$
$$F_{1y} = (15\text{ N})(0.5299)$$
$$F_{1y} = 7.9488\text{ N}$$

The angle is in the 1st quadrant so the *y*-component is positive.

$$F_{2y} = -|\vec{F}_2|\sin 67°$$
$$F_{2y} = -(18\text{ N})(0.9205)$$
$$F_{2y} = -16.5691\text{ N}$$

The angle is in the 4th quadrant so the *y*-component is negative.

From the components, the equilibrating vector is in the second quadrant.

Sum the *x* and *y*-components individually to find the components of the unknown vector.

$$F_{Ax} + F_{Bx} + F_{Cx} = 0.0$$
$$F_{Cx} = 0.0 - F_{Bx} - F_{Bx}$$
$$F_{Cx} = 0.0 - (12.7207\text{ N}) - (7.0332\text{ N})$$
$$F_{Cx} = -19.7539\text{ N}$$

$$F_{Ay} + F_{By} + F_{Cy} = 0.0$$
$$F_{Cy} = 0.0 - F_{Ay} - F_{By}$$
$$F_{Cy} = 0.0 - (7.9488\text{ N}) - (-16.5691\text{ N})$$
$$F_{Cy} = 8.6203\text{ N}$$

Use the Pythagorean Theorem to find the magnitude of $\vec{F}_C$.

$$\left|\vec{F}_C\right|^2 = (F_{Cx})^2 + (F_{Cy})^2$$
$$\left|\vec{F}_C\right|^2 = (-19.7539\text{ N})^2 + (8.6203\text{ N})^2$$
$$\left|\vec{F}_C\right|^2 = 464.526\text{ N}^2$$
$$\left|\vec{F}_C\right| = 21.553\text{ N}$$
$$\left|\vec{F}_C\right| \cong 22\text{ N}$$

Use the tangent function and the magnitudes of the x- and y-components to find the angle θ_3.

$$\tan\theta = \frac{F_{Cy}}{F_{Cx}}$$
$$\tan\theta = \frac{8.6203\text{ N}}{-19.7539\text{ N}}$$
$$\tan\theta = -0.4364$$
$$\theta = \tan^{-1}(-0.4364)$$
$$\theta = -23.5757°$$
$$\theta \cong -24°$$

Caitlin exerts a force of 22 N[W24°N].

Validate the Solution
From a diagram, it is evident that the third vector will be in either the second or third quadrant, so the answer is reasonable.

9. Frame the Problem
- The force of gravity on the traffic light is balanced by the tension force in the cables.
- Draw a free body diagram representing the forces on the traffic light.
- The traffic light is in equilibrium and has no net acceleration in either the vertical or horizontal directions.

Identify the Goal
The force that the cables, $\vec{F}_{Cable}$, exert on the traffic light to prevent if from falling.

Variables and Constants
Identify the Variables

Known	Implied	Unknown
m = 65 kg	$\vec{g} = 9.81\text{ m/s}^2$	$\vec{F}_{Cable}$
θ = 12°		

Develop a Strategy

Apply Newton's second law in the y-direction.

Note from the hint that the tension in the two cables is equal.

Calculations

$$F_{Cable1y} + F_{Cable2y} - mg = 0$$
$$F_{Cable1}\sin 12° + F_{Cable2}\sin 12° = mg$$
$$2F_{Cable}\sin 12° = mg$$
$$F_{Cable} = \frac{mg}{2\sin 12°}$$

Substitute numerical values and solve.

$$F_{Cable} = \frac{(65\text{ kg})(9.81\text{ m/s}^2)}{2(0.2079)}$$
$$F_{Cable} = 1533.46$$
$$F_{Cable} \cong 1.5\times 10^3\text{ N}$$

The tension in the cables is 1.5×10^3 N.
(Note that applying Newton's second law
in the horizontal direction was unnecessary.

Validate the Solution
The tension in the cables is much greater than the weight of the light
(mg = 65 kg $\times$ 9.81 m/s^2 = 638 N), as expected, to keep the light steady and resist additional forces due to wind, etc. Therefore, the answer is reasonable.

Practice Problem Solutions

Student Textbook pages 474–475

10. Frame the Problem

- Sketch a free body diagram of the box on the ramp.
- If the *component* of the *force of gravity* that is *parallel* to the ramp is greater than the *maximum possible friction force*, the box will slide down the ramp. Otherwise it will remain motionless.

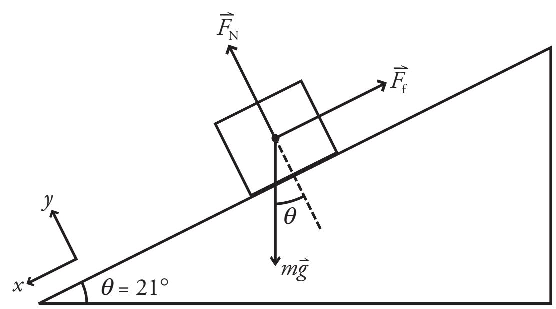
Identify the Goal

(a) Whether the box will slide down the ramp or remain motionless.

(b) The applied force, $\vec{F}_{\text{a}}$, needed to start the box moving, if it was motionless.

Variables and Constants
Identify the Variables

Known	Implied	Unknown
m = 45 kg	$\vec{g} = 9.81 \text{ m/s}^2$	$\vec{F}_{\text{a}}$
θ = 21°		$\vec{F}_{\text{f}}$
μ_{s} = 0.42		



Develop a Strategy
Find the x component of the force of gravity.

Calculations

$$\sin\theta = \frac{F_{\text{gx}}}{|\vec{F}_{\text{g}}|}$$

$$F_{\text{gx}} = |\vec{F}_{\text{g}}|\sin\theta$$

$$F_{\text{gx}} = mg\sin\theta$$

$$F_{\text{gx}} = (45 \text{ kg})(9.81 \text{ m/s}^2)\sin 21°$$

$$F_{\text{gx}} = 158.20 \text{ N}$$

Find the maximum possible force of static friction.

$$F_{f(max)} = \mu_s F_N$$
$$F_{f(max)} = \mu_s mg\cos\theta$$
$$F_{f(max)} = (0.42)(45\text{ kg})(9.81\text{ m/s}^2)\cos 21°$$
$$F_{f(max)} = 173.09\text{ N}$$

(a) Since the force of friction is greater than the *x*-component of the force of gravity, the box will remain motionless.

Take the difference of the forces to determine the required force down the ramp to get the box to start to slide.

$$F_{f(max)} - F_{gx} = 173.09\text{ N} - 153.20\text{ N}$$
$$F_{f(max)} - F_{gx} = 15.0\text{ N}$$

An applied force of 15 N is required to get the box to start to slide.

Validate the Solution

The ramp has only a shallow tilt, so it is reasonable that the box does not slide. It is also reasonable that a small nudge of 15 N will get it started.

11. Frame the Problem

- Sketch a free body diagram of the container on the ramp. (It's the same as in the previous question.)
- When the *component* of the *force of gravity* that is *parallel* to the ramp becomes greater than the *maximum possible friction force*, the container will begin to slide down the ramp.
- When the container begins sliding, Newton's second law can be applied to determine the *acceleration* down the ramp.

Identify the Goal

(a) The angle, θ, of the ramp at which the container just starts to slide.

(b) The acceleration, a_x , of the container just after it started to slide.

Variables and Constants

Identify the Variables

Known	Implied	Unknown
m = 61 kg	$\vec{g} = 9.81\text{ m/s}^2$	θ
μ_s = 0.37		a_x
μ_k = 0.18		

Develop a Strategy / **Calculations**

Find the x component of the force of gravity.

$$\sin\theta = \frac{F_{gx}}{|\vec{F}_g|}$$
$$F_{gx} = |\vec{F}_g|\sin\theta$$
$$F_{gx} = mg\sin\theta$$

Find the maximum possible force of static friction.

$$F_{f(max)} = \mu_s F_N$$
$$F_{f(max)} = \mu_s mg\cos\theta$$

The container will begin to slide when $F_{gx} > F_{f(max)}$. So, to find the angle, set the ratio of these to 1 and solve for the angle.

$$\frac{F_{gx}}{F_{f(max)}} = 1 = \frac{mg\sin\theta}{\mu_s mg\cos\theta}$$

$$\frac{\tan\theta}{\mu_s} = 1$$

$$\tan\theta = \mu_s$$

$$\tan\theta = 0.37$$

$$\theta = \tan^{-1}(0.37)$$

$$\theta = 20.3°$$

(a) The container will begin to slide when the ramp has an angle of 20.3°.

As soon as the container begins to slide, the coefficient of kinetic friction replaces the coefficient of static friction. Find the acceleration in the *x* direction (down the ramp) by applying Newton's second law to the components of force in the *x* direction.

The component of the gravitational force and the frictional force act in opposite directions so they have opposite signs.

$$mg\sin\theta - \mu_k F_N = ma_x$$

$$mg\sin\theta - \mu_k mg\cos\theta = ma_x$$

$$a_x = g(\sin\theta - \mu_k\cos\theta)$$

$$a_x = 9.81\ \text{m/s}^2(\sin 20.3° - 0.18\cos 20.3°)$$

$$a_x = 1.747\ \text{m/s}^2$$

$$a_x \cong 1.7\ \text{m/s}^2$$

(b) After the container begins to slide, its acceleration would be 1.7 m/s^2 down the ramp.

Validate the Solution

The angle for the ramp is reasonable (and similar to those given in previous questions). The value obtained for the acceleration is much less than the acceleration due to gravity (or in free fall), so it is reasonable.

12. Frame the Problem

- Sketch a free body diagram of the container on the ramp.
- By applying Newton's second law to the components of the forces in the *x* and *y*-directions, the unknown forces can be determined from the known forces.
- Pushing the container at constant velocity implies that the acceleration is zero.

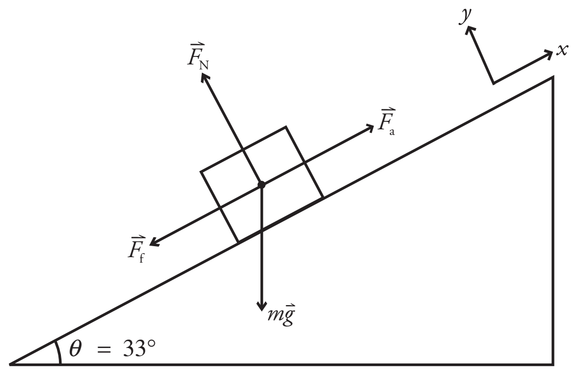
Identify the Goal

The applied force, F_a, required the push the container up the ramp at a constant velocity.

Variables and Constants

Identify the Variables

Known	Implied	Unknown
$m = 55$ kg	$\vec{g} = 9.81\ \text{m/s}^2$	F_a
$\mu_k = 0.18$	$a_x = 0$	
$\theta = 33°$		



Develop a Strategy

Apply Newton's second law in the *y*-direction.

$$F_N = mg\cos\theta$$

Apply Newton's second law in the *x*-direction, noting that the acceleration in this direction is zero.

$$-F_{gx} - F_f + F_a = ma_x = 0$$
$$F_a = F_f + F_{gx}$$
$$F_a = \mu_k F_N + mg\sin\theta$$

Substitute the normal force and expand the terms.

$$F_a = \mu_k mg\cos\theta + mg\sin\theta$$
$$F_a = mg(\mu_k\cos\theta + \sin\theta)$$

Substitute numerical values and solve.

$$F_a = (55\text{ kg})(9.81\text{ m/s}^2)(0.23\cos 33° + \sin 33°)$$
$$F_a = 397.94\text{ N}$$
$$F_a \cong 4.0\times 10^2\text{ N}$$

An applied force of 4.0×10^2 N is required.

Validate the Solution

The force required is slightly less than the weight of the container ($mg = 55\text{ kg} \times 9.81\text{ m/s}^2 = 540\text{ N}$). Because the ramp is relatively steep, a significant force is required to move the container along at a constant velocity, so the answer is reasonable.

For comparison, moving the container at constant velocity along the ground would require a force, $F_a = \mu mg = 0.23 \times 55\text{ kg} \times 9.81\text{ m/s}^2 = 124\text{ N}$, so, clearly, the steepness of the ramp makes it necessary to apply a much larger force.

13. Frame the Problem

- Sketch a free body diagram of the crate on the ramp.
- By applying Newton's second law to the components of the forces in the *x* and *y*-directions, the unknown forces can be determined from the known forces.
- In order to start the crate moving, the *coefficient of static friction* is required.

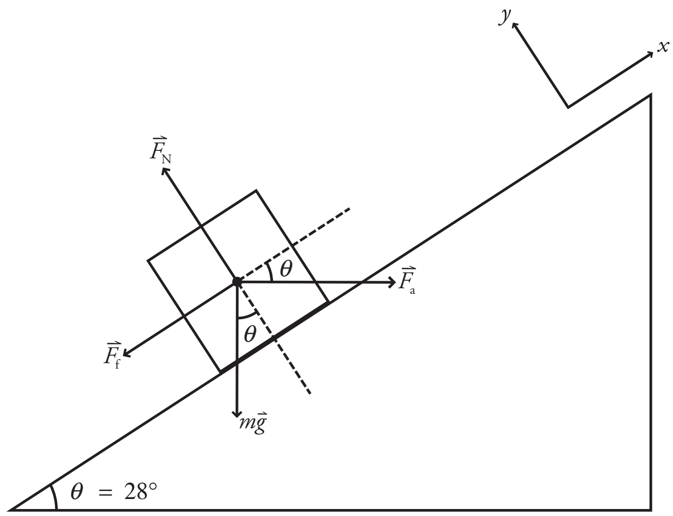
Identify the Goal

(a) The applied force, F_a, required the start the crate moving up the ramp, when pushing parallel to the ground.

(b) The applied force, F_a, required the start the crate moving up the ramp, when pushing parallel to the ramp.

Variables and Constants
Identify the Variables

Known	Implied	Unknown
$m = 85$ kg	$\vec{g} = 9.81\ \text{m/s}^2$	F_a
$\mu_s = 0.46$		
$\theta = 28°$		



Develop a Strategy

Calculations

Apply Newton's second law in the *y*-direction. Note the hint, which says that a component of the applied force is perpendicular to the ramp and thus contributes to the normal force.

$$F_N - F_{gy} - F_{ay} = 0$$
$$F_N = F_{gy} + F_{ay}$$
$$F_N = mg\cos\theta + F_a\sin\theta$$

Apply Newton's second law in the *x*-direction, noting that the acceleration in this direction is zero.

Substitute the normal force and expand the terms.

$$F_{ax} - F_f - F_{gx} = ma_x = 0$$
$$F_a\cos\theta - \mu_s F_N - mg\sin\theta = 0$$
$$F_a\cos\theta - \mu_s(mg\cos\theta + F_a\sin\theta) - mg\sin\theta = 0$$
$$F_a\cos\theta - \mu_s mg\cos\theta - \mu_s F_a\sin\theta - mg\sin\theta = 0$$
$$F_a\cos\theta - \mu_s F_a\sin\theta = \mu_s mg\cos\theta + mg\sin\theta$$

Substitute numerical values and solve.

$$F_a = \frac{\mu_s mg\cos\theta + mg\sin\theta}{\cos\theta - \mu_s\sin\theta}$$
$$F_a = \frac{(0.46)(85\text{ kg})(9.81\text{ m/s}^2)\cos 28° + (85\text{ kg})(9.81\text{ m/s}^2)\sin 28°}{\cos 28° + (0.46)\sin 28°}$$
$$F_a = 1094.68\text{ N}$$
$$F_a \cong 1.1\times 10^3\text{ N}$$

(a) An applied force of 1.1×10^3 N is required.

When the worker pushes parallel to the ramp, the applied force is parallel to the ramp. So, the terms in the above equation (derived from the x-components) that contain $F_a\cos\theta$ or $F_a\sin\theta$ should be replaced by F_a and 0, respectively, because the pushing angle is now zero.

Effectively, this sets the denominator in the above equation to 1.

Substitute numerical values and solve.

$$F_a = \mu_s mg\cos\theta + mg\sin\theta$$
$$F_a = (0.46)(85\text{ kg})(9.81\text{ m/s}^2)\cos 28° + (85\text{ kg})(9.81\text{ m/s}^2)\sin 28°$$
$$F_a = 730.14\text{ N}$$
$$F_a \cong 7.3\times 10^2\text{ N}$$

(b) To start the crate moving up the ramp by pushing parallel to the ramp requires a force of 7.3×10^2 N.

Validate the Solution

The units work out to be newtons in each case, as expected. By pushing at a more appropriate angle in the second case, less force is required, so the answer is reasonable.

Practice Problem Solutions

Student Textbook page 478

14. Conceptualize the Problem

- Begin framing the problem by drawing a *free-body diagram* of the *forces* acting on the girl.
- The *tension* of the rope pulls her up, while the *force of gravity* pulls her down.
- Let "up" be *positive* and "down" be *negative*.
- Because the girl *accelerates* downward, it is expected that the sign of the *acceleration* will be *negative*.
- The *acceleration* can be found by applying *Newton's second law*.

Identify the Goal

The acceleration, $\vec{a}$, of the child as she is being lowered

Identify the Variables

Known	Implied	Unknown
$m = 32$ kg	$\vec{g} = -9.81\text{ m/s}^2[\text{up}]$	$\vec{a}$
$\vec{F}_t = 253\text{ N[up]}$		

Develop a Strategy

Apply Newton's second law to the free-body diagram.

Substitute and solve.

The child has an acceleration of about $-1.9\ \text{m/s}^2[\text{up}]$.

Calculations

$\vec{F}_t + \vec{F}_g = ma$

$\vec{a} = \dfrac{\vec{F}_t + \vec{F}_g}{m} = \dfrac{\vec{F}_t + (m\vec{g})}{m}$

$\vec{a} = \dfrac{253\ \text{N[up]} + (32\ \text{kg})(-9.81\ \text{m/s}^2[\text{up}])}{32\ \text{kg}}$

$\vec{a} = -1.904\ \text{m/s}^2[\text{up}]$

$\vec{a} \cong -1.9\ \text{m/s}^2[\text{up}]$

Validate the Solution

The child is being lowered from a rope held by her parent, so it is expected that her acceleration will be downward, and much less than the acceleration due to gravity, which it is.

15. Solution is similar to Practice Problem 14.
The climber has an acceleration of $2.5\ \text{m/s}^2[\text{down}]$.

16. Conceptualize the Problem

- Begin framing the problem by drawing a *free-body diagram* of the forces acting on the mass on the scale.
- The *forces* acting on the mass are *gravity* ($\vec{F}_g$), which is downwards, and the *normal force* of the scale, which is upwards.
- According to Newton's third law, when the mass exerts a force (F_{MS}) on the scale, it exerts an equal and opposite force (F_{SM}) on the mass. Therefore, the reading on the scale is the same as the force that the mass exerts on the scale. In the same way, the tension in the hoist rope will also be the same.
- The motion is in one direction so let "up" be positive and "down" be negative.
- Because the motion is in one dimension, perform calculations with magnitudes only.
- Also, because the mass is just starting to move, the sign of the acceleration will indicate the direction of the motion (the mass is accelerating, not decelerating).
- Apply Newton's second law to find the acceleration of the mass.

Identify the Goals

(a) the direction of motion.

(b) the acceleration, a, of the mass.

(c) the tension, F_T, of the hoist rope.

Identify the Variables

Known	Implied	Unknown
$m = 10.0\ \text{kg}$	$g = 9.81\ \text{m/s}^2$	$\vec{F}_{MS}$
$\vec{F}_{SM} = 87\ \text{N}$		$\vec{F}_g$
		$\vec{F}_T$
		a

Develop a Strategy

Apply Newton's second law to the free-body diagram.

Substitute and solve.

Calculations

$$F_{SM} + F_g = ma$$

$$F_{SM} + (-mg) = ma$$

$$\frac{F_{SM} - mg}{m} = a$$

$$a = \frac{87\ \text{N} - (10.0\ \text{kg})(9.81\ \text{m/s}^2)}{10.0\ \text{kg}}$$

$$a = -1.11\ \text{m/s}^2$$

$$a \cong -1.1\ \text{m/s}^2$$

(a) The acceleration is negative, so the motion is downwards.

(b) The acceleration of the mass is $-1.1\ \text{m/s}^2$[up].

(c) From Newton's third law, the tension in the hoist rope will be the same as the force the mass exerts on the scale, or 87 N.

Validate the Solution

Because of the negative sign of the acceleration (i.e. directed downwards), the reading on the scale is slightly less than it would be if the hoist had zero acceleration. In that case, the scale would read, $(10.0\ \text{kg})(9.81\ \text{m/s}^2) = 98.1\ \text{N}$. This agrees with the experience of riding in an elevator: as it starts travelling downwards, you feel lighter.

17. Solution is similar to Practice Problem 16.
The tension of the backpack strap is about 1.7×10^2 N[up].

18. Solution is similar to Practice Problem 16.
The elevator has a maximum acceleration of about $1.8\ \text{m/s}^2$[up].

Practice Problem Solutions

Student Textbook page 485

19. Conceptualize the Problem

- Begin framing the problem by drawing *free-body diagrams.* Draw one diagram of the system moving as a unit and diagrams of each of the two individual objects.
- Let the *negative* direction point from the centre to the 3.8 kg mass and the *positive* direction point from the centre to the 4.2 kg mass. (If the rope was stretched out horizontally, the 3.8 kg mass would be to the *left* and the 4.2 kg mass would be to the *right.*) That is, "down" is *positive* for the 4.2 kg mass. See Figure 10.12 in the Student Text, and the Sample Problem, Motion of Connected Objects.
- Both objects move with the same *acceleration.* The *heavier* mass will move downwards and the *lighter* mass will move upwards.
- The *force of gravity* acts on both objects.
- The *tension* is constant throughout the rope.
- The rope exerts a *force* of equal magnitude and opposite direction on each object.
- When you isolate the individual objects, the *tension* in the rope is one of the forces acting on the object.
- *Newton's second law* applies to the combination of the two objects and to each individual object.

Problem Tip

Vectors should always be written with both a magnitude and a direction. In problems such as the Atwood machine or Fletcher's trolley, the directions are usually defined from the centre of the rope and by considering the rope as stretched out. Thus, students may have two pictures in their minds: the masses hanging on either side of the pulley, and the rope stretched horizontally. Depending on which picture they have, they may use terms like "right" and "left", or "up" and "down" to define the directions of the vectors, which can be confusing — for example, talking about gravity as acting to the right! Therefore, it is perhaps simplest to define the coordinate system initially, and then to work with the magnitudes of the vectors in the calculations.

Identify the Goal

The acceleration, $\vec{a}$, of the two objects

The tension, $\vec{F}_T$, in the rope

Identify the Variables

Known	Implied	Unknown
$m_1 = 3.8$ kg	$g = 9.81$ m/s^2	$\vec{F}_T$
$m_2 = 4.2$ kg		$\vec{a}$
		$\vec{F}_{g1}$
		$\vec{F}_{g2}$

Develop a Strategy

Apply Newton's second law to the combination of masses to find the acceleration.
As the motion can be considered to be in one dimension (if the rope is stretched out), the calculations can be done with magnitudes.

Calculations

$$F = ma$$
$$F_{g1} + F_{g2} = (m_1 + m_2)a$$
$$-m_1 g + m_2 g = (m_1 + m_2)a$$
$$a = \frac{(m_2 - m_1)g}{m_1 + m_2}$$

Substitute and solve.

$$a = \frac{(4.2\text{ kg} - 3.8\text{ kg})(9.81\text{ m/s}^2)}{4.2\text{ kg} + 3.8\text{ kg}}$$
$$a = 0.4905\text{ m/s}^2$$
$$a \cong 0.49\text{ m/s}^2$$

The acceleration of the masses is about 0.49 m/s^2. The positive sign indicates the system accelerates in a clockwise direction.

Apply Newton's second law to m_1 and solve for tension.

$$F = ma$$
$$F_{g1} + F_T = m_1 a$$
$$-m_1 g + F_T = m_1 a$$
$$F_T = m_1 g + m_1 a = m_1(g + a)$$

Substitute and solve.

$$F_T = (3.8\text{ kg})(9.81\text{ m/s}^2 + 0.4905\text{ m/s}^2)$$
$$F_T = 39.14\text{ N}$$
$$F_T \cong 39\text{ N}$$

The tension in the rope is about 39 N. The positive sign indicates the tension on m_1 acts to the right.

Validate the Solution

You can test your solution by applying Newton's second law to the second mass.

$$F = ma$$
$$F_{g2} + F_T = m_2 a$$
$$F_T = m_2 a - m_2 g$$
$$F_T = m_2(a - g)$$
$$F_T = (4.2\ \text{kg})(0.4905\ \text{m/s}^2 - 9.81\ \text{m/s}^2)$$
$$F_T = -39.14\ \text{N}$$
$$F_T \cong -39\ \text{N}$$

The magnitudes of the tensions calculated independently from each of the masses agree. Also, notice that the application of Newton's second law correctly gave the direction for the force on the second mass (to the left).

20. Solution is similar to Practice Problem 19.
The mass of the second object is about 14 kg.
The tension in the rope is about 75 N[to the right].

21. Solution is similar to Practice Problem 19.
The acceleration of the system is about $1.6\ \text{m/s}^2$[clockwise].
The mass of the second object is about 62 kg.

22. Solution is similar to Practice Problem 19.
The applied force is about 17 N.

23. Solution is similar to Practice Problem 19.
Both gymnasts accelerate upwards at $1.0\ \text{m/s}^2$.

Practice Problem Solutions

Student Textbook pages 488–489

24. Conceptualize the Problem

- Make a simplified diagram of the connected masses and assign forces.
- Visualize the light string in a straight configuration.
- Sketch *free-body diagrams* of the *forces* acting on each object and of the *forces* acting on the combined objects.
- The trolley experiences a *horizontal force* due to the *tension* in the string and *vertical forces of gravity* and the *normal force*, which act opposite to one another. See Figure 10.13 in the Student Text, and the Sample Problem, Connected Objects.
- The suspended mass experiences the *force of gravity* and the *normal force*, which act opposite to one another.
- The *force* causing the *acceleration* of both masses is the *force of gravity* acting on the suspended mass.
- *Newton's second law* applies to the combined masses and to each individual mass.
- Let left be the *negative* direction and right be the *positive* direction.

Identify the Goal

(a) The tension, $\vec{F}_T$, in the string when the suspended mass is released

(b) The acceleration, $\vec{a}$, of the trolley

Identify the Variables

Known	Implied	Unknown
$m_1 = 1.90$ kg	$g = 9.81$ m/s^2	$\vec{F}_N$ $\vec{F}_T$
$m_2 = 0.500$ kg		$\vec{a}$
		$\vec{F}_{g1}$
		$\vec{F}_{g2}$

Develop a Strategy

Apply Newton's second law to the combined masses to find the acceleration. As the motion can be considered to be in one dimension (if the string is stretched out), the calculations can be done with magnitudes (see also the Problem Tip in Practice Problem 19).

Substitute and solve.

Calculations

$$\vec{F}_{net} = m\vec{a}$$
$$F_{g2} = (m_1 + m_2)a$$
$$m_2 g = (m_1 + m_2)a$$
$$a = \frac{m_2 g}{m_1 + m_2}$$

$$a = \frac{(0.500 \text{ kg})(9.81 \text{ m/s}^2)}{1.90 \text{ kg} + 0.500 \text{ kg}}$$
$$a = 2.0438 \text{ m/s}^2$$
$$a \cong 2.04 \text{ m/s}^2$$

(a) The tension in the string is 3.88 N. Because the sign is positive, the tension acting on m_1 is to the right.

(b) The acceleration is about 2.04 m/s^2, to the right.
Apply Newton's second law to the trolley (mass 1) to find the tension. Use magnitudes in the calculation.

$$\vec{F}_{net} = m\vec{a}$$
$$F_T = m_1 a$$
$$F_T = (1.90 \text{ kg})(2.0438 \text{ m/s}^2)$$
$$F_T = 3.88 \text{ N}$$

Validate the Solution

The acceleration of the combined masses is less than 9.81 m/s^2, which is reasonable since only part of the mass is subject to unbalanced gravitational forces. The tension can be verified by applying Newton's second law to the suspended mass.

$$\vec{F}_{net} = ma$$
$$F_T + F_{g2} = m_2 a$$
$$F_T = m_2 a - m_2 g$$
$$F_T = m_2(a - g)$$
$$F_T = (0.500 \text{ kg})(2.0438 \text{ m/s}^2 - 9.81 \text{ m/s}^2)$$
$$F_T = -3.88 \text{ N}$$

The negative sign indicates that the tension on the suspended mass acts upwards.

25. Solution is similar to Practice Problem 24.
The glider travels to the end of the track in 0.67 s.

26. Frame the Problem

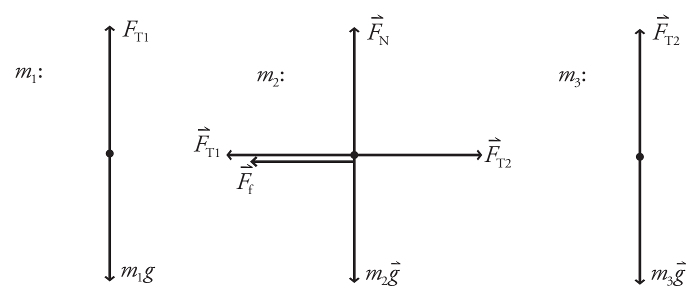
- Sketch the apparatus in its correct configuration with forces added. Add a coordinate system. In this case, let the direction to the right be positive.
- Then sketch the apparatus with the string in a straight line.
- Make free body diagrams of the bodies individually.
- Only block 2 is experiencing *friction* with the table.

- Due to the different masses on either ends of the system, the *tensions* in strings are not necessarily the same.
- Despite the different *tensions*, the system will move with *uniform acceleration*.
- The masses on either ends of the system feel the *force of gravity* and the *tensions* in the strings.
- Apply Newton's second law to each free body diagram individually in both the *x*- and *y*-directions. Examine the set of equations and determine how best to solve for the unknowns.
- Once the acceleration is found, use a kinematical equation to find the time it takes the mass to fall.

Identify the Goal
The time interval, Δt, it takes for the mass, m_3, to reach the ground once the system starts moving.

Variables and Constants
Identify the Variables

Known	Implied	Unknown
m_1 = 228 g = 0.228 kg	$\vec{g} = 9.81 \text{ m/s}^2$	a
m_2 = 615 g = 0.615 kg		Δt
m_3 = 455 g = 0.455 kg		
μ_k = 0.26		
Δd = 1.95 m		



Develop a Strategy	Calculations
Apply Newton's second law to each free body diagram, in both the *x*- and *y*-directions.	
For m_1. (Only the *x*-direction is required.)	$F_{T1} - m_1 g = m_1 a \quad (1)$
For m_2. Note the acceleration in the *y*-direction is zero.	*x*: $F_{T2} - F_{T1} - F_f = m_2 a$ or, $F_{T2} - F_{T1} - \mu_k F_N = m_2 a \quad (2a)$ y: $F_N - m_2 g = 0 \quad (2b)$
For m_3. (Only the *x*-direction is required.)	$m_3 g - F_{T2} = m_3 a \quad (3)$

By inspection, equations (1), (2b) and (3) can be rearranged and substituted into equation (2a).

$$F_{T2} - F_{T1} - \mu_k F_N = m_2 a \qquad (2a)$$
$$(m_3 g - m_3 a) - F_{T1} - \mu_k F_N = m_2 a$$
$$(m_3 g - m_3 a) - (m_1 a + m_1 g) - \mu_k F_N = m_2 a$$
$$(m_3 g - m_3 a) - (m_1 a + m_1 g) - \mu_k (m_2 g) = m_2 a$$
$$m_3 g - m_3 a - m_1 a - m_1 g - \mu_k m_2 g = m_2 a$$

Collect like terms on each side of the equation.

$$m_3 g - m_1 g - \mu_k m_2 g = m_2 a + m_3 a + m_1 a$$
$$(m_1 + m_2 + m_3) a = (m_3 - m_1 - \mu_k m_2) g$$

Solve for the acceleration.

$$a = \frac{(m_3 - m_1 - \mu_k m_2) g}{(m_1 + m_2 + m_3)}$$

Substitute numerical values and solve.

$$a = \frac{(0.455\ \text{kg} - 0.228\ \text{kg} - (0.26)(0.615\ \text{kg})}{0.228\ \text{kg} + 0.615\ \text{kg} + 0.455\ \text{kg}} \times 9.81\ \text{m/s}^2$$
$$a = 0.5071\ \text{m/s}^2$$
$$a \cong 0.507\ \text{m/s}^2$$

The system accelerates at 0.507 m/s^2.

Use a kinematics equation that relates the distance, acceleration and time interval to find the time interval.

$$\Delta d = \tfrac{1}{2} a t^2$$
$$t = \sqrt{\frac{2\Delta d}{a}}$$
$$t = \sqrt{\frac{2(1.95\ \text{m})}{0.5071\ \text{m/s}^2}}$$
$$t = 2.773\ \text{s}$$
$$t \cong 2.77\ \text{s}$$

Once the system starts moving, it takes 2.77 s for the third mass to hit the floor.

Validate the Solution

The units work out properly in both cases. The time interval of almost 3 s to fall about 2 m is very reasonable.

27. Frame the Problem

- Sketch the apparatus in its correct configuration with forces added. Add a coordinate system. In this case, let the direction to the right be positive.
- Then sketch the apparatus with the string in a straight line.
- Make free body diagrams of the bodies individually and as one system.
- Block 1 is experiencing *friction* with the ramp.
- Both a *component* of the *gravitational force* and a *frictional force* on the block are creating a force in the *negative* direction.
- For m_2, the *tension* operates in the *negative* direction and the *gravitational force* operates in the positive direction.
- Find the *acceleration* and use the definition of *acceleration* to find the *velocity* (or speed).

Identify the Goal

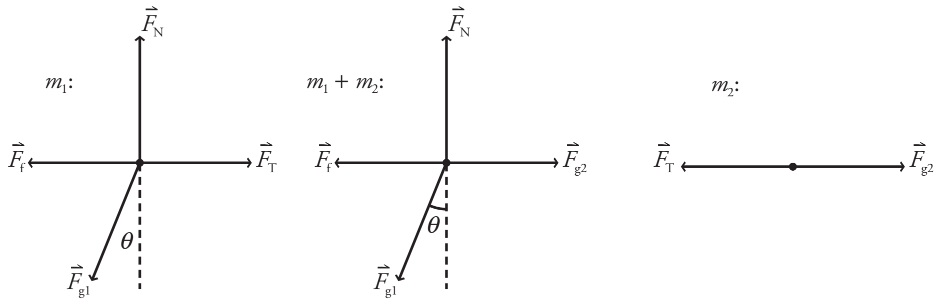
(a) The speed, v, of the masses 2.5 s after they start moving.

(b) The tension in the string, F_T, while they are moving.

Variables and Constants

Identify the Variables

Known	Implied	Unknown
$m_1 = 145\ \text{g} = 0.145\ \text{kg}$	$\vec{g} = 9.81\ \text{m/s}^2$	$\vec{a}$
$m_2 = 85\ \text{g} = 0.085\ \text{kg}$	$v_\text{i} = 0.0\ \text{m/s}$	v
$\mu_\text{k} = 0.18$		$\lvert\vec{F}_\text{T}\rvert$
$\Delta t = 2.5\ \text{s}$		



Develop a Strategy

To find the frictional force, you need the normal force.

Apply Newton's second law in the direction perpendicular to the plane.

Calculations

$$F_\perp = ma_\perp = 0$$
$$F_\text{N} - F_{\text{g1}\perp} = 0$$
$$F_\text{N} = F_{\text{g1}\perp}$$
$$F_\text{N} = m_1 g\cos\theta$$

Since the two masses have the same acceleration, define the system as the combination of the masses.

Find the acceleration from Newton's second law in the direction of the string when it is drawn horizontal. Call this the *x* direction. Substitute in the normal force from the above, and expand terms.

Solve for the acceleration.

Substitute numerical values and solve.

$$F_\text{x} = ma_\text{x}$$
$$F_\text{g2x} - F_\text{g1x} - F_\text{f} = (m_1 + m_2)a_\text{x}$$
$$\text{m}_2 g - m_1 g\sin\theta - \mu_\text{k}F_\text{N} = (m_1 + m_2)a_\text{x}$$
$$a_\text{x} = \frac{\text{m}_2 g - m_1 g\sin\theta - \mu_\text{k}(m_1 g\sin\theta)}{(m_1 + m_2)}$$
$$a_\text{x} = \frac{\text{m}_2 - m_1\sin\theta - \mu_\text{k}m_1\sin\theta}{(m_1 + m_2)}g$$
$$a_\text{x} = \frac{0.085\ \text{kg} - (0.145)\sin 22° - (0.18)(0.145\ \text{kg})\cos 22°}{(0.145\ \text{kg} + 0.085\ \text{kg})} \times 9.81\,\text{m/s}^2$$
$$a_\text{x} = 0.2765\ \text{m/s}^2$$

Use the definition of acceleration to find the velocity.

$$a_x = \frac{\Delta v}{\Delta t}$$

The initial velocity is zero.

$$\Delta v = (v - v_i) = a_x \Delta t$$
$$v = (0.2765\ \text{m/s}^2)(2.5\ \text{s})$$
$$v = 0.6912\ \text{m/s}$$
$$v \cong 0.69\ \text{m/s}$$

(a) The speed of the masses after they start to move will be 0.69 m/s.

By examining the free body diagram for the mass m_2, the tension can be found.

$$F_{g2} - F_T = m_2 a_x$$
$$F_T = F_{g2} - m_2 a_x$$

Rearrange and solve for the tension.

$$F_T = m_2 g - m_2 a_x$$
$$F_T = m_2(g - a_x)$$

Substitute numerical values and solve.

$$F_T = (0.085\ \text{kg})(9.81\ \text{m/s}^2 - 0.2765\ \text{m/s}^2)$$
$$F_T = 0.810\ \text{N}$$
$$F_T \cong 0.81\ \text{N}$$

(b) The tension in the string while they are moving is 0.81 N.

Validate the Solution

The units work out properly in both cases. The speed is equivalent to 69 cm/s which seems reasonable considering that the ramp is not steep so the free hanging mass pulls it along quickly.

28. Frame the Problem

- Sketch the apparatus in its correct configuration with forces added. Add a coordinate system. In this case, let the direction to the right be positive.
- Then sketch the apparatus with the string in a straight line.
- Make free body diagrams of the bodies individually and as one system.
- Block 1 is experiencing *friction* with the ramp.
- Both a *component* of the *gravitational force* and a *frictional force* on the block are creating a force in the *negative* direction.
- For m_2, the *tension* operates in the *negative* direction and the *gravitational force* operates in the positive direction.

Identify the Goal

(a) The force to apply, F_a, to make the objects start to move.

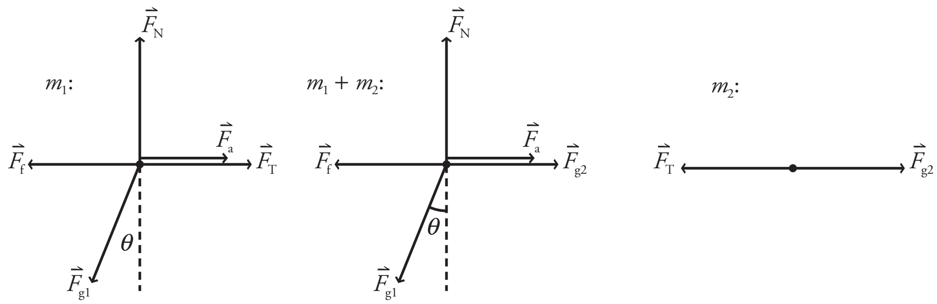
(b) The acceleration, a, after the objects start to move.

(c) The tension, F_T, in the string when the objects are moving.

Variables and Constants

Identify the Variables

Known	Implied	Unknown
m_1 = 725 g = 0.725 kg	$\vec{g} = 9.81\ \text{m/s}^2$	F_a
m_2 = 595 g = 0.595 kg		$\vec{a}$
μ_k = 0.12		F_T
μ_s = 0.47		



Develop a Strategy | **Calculations**

To find the frictional force, you need the normal force.

Apply Newton's second law in the direction perpendicular to the plane.

$$F_\perp = ma_\perp = 0$$
$$F_N - F_{g1\perp} = 0$$
$$F_N = F_{g1\perp}$$

Save this equation until it is needed.

$$F_N = m_1 g\cos\theta$$

Since the two masses have the same acceleration, define the system as the combination of the masses. Note that until the system begins to move, the acceleration is zero.

Find the applied force from Newton's second law in the direction of the string when it is drawn horizontal. Call this the x direction. Substitute in the normal force from the above, and expand terms.

$$F_x = ma_x$$
$$F_a + F_{g2x} - F_{g1x} - F_f = (m_1 + m_2)a_x = 0$$
$$F_a + m_2 g - m_1 g\sin\theta - \mu_s F_N = 0$$

Solve for the applied force.

$$F_a = -m_2 g + m_1 g\sin\theta + \mu_s(m_1 g\cos\theta)$$
$$F_a = (-m_2 + m_1\sin\theta + \mu_s m_1\cos\theta)g$$

Substitute numerical values and solve.

$$F_a = (0.595\text{ kg} + (0.725\text{ kg})\sin 34° + (0.47)(0.725\text{ kg})\cos 34°) \times 9.81\,\text{m/s}^2$$
$$F_a = 0.9114\text{ N}$$
$$F_a \cong 0.91\text{ N}$$

(a) The applied force is 0.91 N.

To calculate the acceleration after the system starts to move, assume the applied force is now zero and proceed as above.

$$F_x = ma_x$$
$$F_a + F_{g2x} - F_{g1x} - F_f = (m_1 + m_2)a_x$$
$$0 + m_2 g - m_1 g\sin\theta - \mu_k F_N = (m_1 + m_2)a_x$$
$$m_2 g - m_1 g\sin\theta - \mu_k(m_1 g\cos\theta) = (m_1 + m_2)a_x$$
$$a_x = \frac{m_2 g - m_1 g\sin\theta - \mu_k m_1 g\cos\theta}{(m_1 + m_2)}$$
$$a_x = \frac{m_2 - m_1\sin\theta - \mu_k m_1\cos\theta}{(m_1 + m_2)}g$$
$$a_x = \frac{(0.595\text{ kg}) - (0.12)(0.725\text{ kg})\cos 34° - (0.725\text{ kg})\sin 34°}{(0.725\text{ kg} + 0.595\text{ kg})}$$
$$a_x = 0.873\text{ m/s}^2$$
$$a_x \cong 0.87\text{ m/s}^2$$

(b) The acceleration of the masses after they start to move will be 0.87 m/s².

By examining the free body diagram for the mass m_2, the tension can be found.

$F_{g2} - F_T = m_2 a_x$

$F_T = F_{g2} - m_2 a_x$

Rearrange and solve for the tension.

$F_T = m_2 g - m_2 a_x$

$F_T = m_2(g - a_x)$

Substitute numerical values and solve.

$F_T = (0.595 \text{ kg})(9.81 \text{ m/s}^2 - 0.873 \text{ m/s}^2)$

$F_T = 5.318 \text{ N}$

$F_T \cong 5.3 \text{ N}$

(c) The tension in the string while they are moving is 5.3 N.

Validate the Solution
The units work out properly in each case. The ramp is relatively steep, so a low value for the acceleration is expected. The acceleration is less than 1/10 of the acceleration due to gravity (free fall), which seems reasonable.

Practice Problem Solutions

Student Textbook page 495

29. Frame the Problem
- Sketch the situation, indicating the direction of the force.
- Use the equation for *torque* to determine the magnitude of the torque.

Identify the Goal
The torque, τ, exerted by your biceps muscle.

Variables and Constants
Identify the Variables

Known	Unknown
$r_\perp$ = 0.052 m	τ
F = 1250 N	

Develop a Strategy
The lever arm is the distance from the hinge of the elbow to where the biceps attaches.

Simply apply the equation for torque.

The torque is 65 N·m.

Calculations

$\tau = r_\perp F$

$\tau = (0.052 \text{ m})(1250 \text{ N})$

$\tau = 65 \text{ N} \cdot \text{m}$

Validate the Solution
The units are newton metres, the correct unit for torque.

30. Frame the Problem
- Sketch the situation, indicating the direction of the force.
- Use the equation for *torque* to determine the magnitude of the torque.

Identify the Goal
The torque, τ, exerted by the painter's weight on the ladder.

Variables and Constants
Identify the Variables

Known	Unknown
$r_{\perp} = 0.052$ m	τ
$F = 1250$ N	

Develop a Strategy
The lever arm is the distance from the hinge of the elbow to where the biceps attaches.

Simply apply the equation for torque.

The torque is 65 N·m.

Calculations

$$\tau = r_{\perp}F$$
$$\tau = (0.052\ \text{m})(1250\ \text{N})$$
$$\tau = 65\ \text{N}\cdot\text{m}$$

Validate the Solution
The units are newton metres, the correct unit for torque.

Practice Problem Solutions

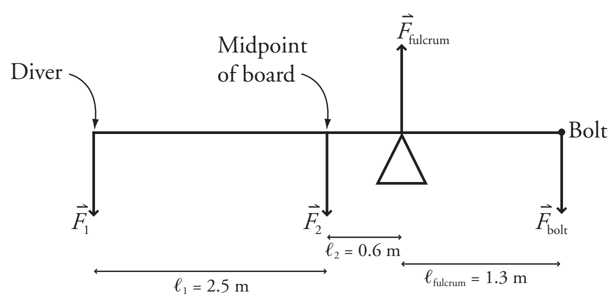
Student Textbook page 501

31. Frame the Problem
- Sketch the situation, indicating the directions of the forces.
- Let the axis of rotation be the fulcrum. This will simplify the calculations (compared to choosing the bolt as the axis of rotation).
- Use the equation for *rotational equilibrium* to find the *force* the bolt exerts on the board.

Identify the Goal
The force, F_{bolt}, the bolt must exert on the diving board to hold it in place.

Variables and Constants
Identify the Variables

Known	Implied	Unknown
$m_{diver} = 54$ kg	$g = 9.81$ m/s^2	F_{bolt}
$m_{board} = 25$ kg		
$l = 3.8$ m		
$l_{fulcrum} = 1.3$ m		



Develop a Strategy
Begin by writing the equation for rotational equilibrium, using the fulcrum as the axis.

Calculations

The lever arm for the diver is then
l_1 = 3.8 m – 1.3 m = 2.5 m.

The lever arm for the board, assuming its weight acts at its midpoint, is,
l_2 = 3.8 m/2 – 1.3 m = 0.6 m.

Because the lever arm for the force acting at the fulcrum is zero, this force does not come into the calculation.

Substitute numerical values and solve.

The bolt exerts a force of 1.1×10^3 N on the board.

Note that at this point the problem is solved and the equations of translational equilibrium do not need to be applied.

$$-F_1 l_1 - F_2 l_2 + F_{bolt} l_{fulcrum} = 0$$

$$F_{bolt} l_{fulcrum} = F_1 l_1 + F_2 l_2$$

$$F_{bolt} = \frac{F_1 l_1 + F_2 l_2}{l_{fulcrum}}$$

$$F_{bolt} = \frac{m_1 g l_1 + m_2 g l_2}{l_{fulcrum}}$$

$$F_{bolt} = \frac{(54\ \text{kg})(9.81\ \text{m/s}^2)(2.5\ \text{m}) + (25\ \text{kg})(9.81\ \text{m/s}^2)(0.6\ \text{m})}{1.3\ \text{m}}$$

$$F_{bolt} = 1131.92\ \text{N}$$

$$F_{bolt} \cong 1.1 \times 10^3\ \text{N}$$

Validate the Solution

The units are newtons, as required. The bolt exerts a force greater than the weight of the diver and the board, as expected. The value seems reasonable.

32. Frame the Problem

- Sketch the situation, indicating the directions of the forces.
- Let the axis of rotation be the elbow joint.
- Use the equation for *rotational equilibrium* to find the *force* the biceps exerts on the forearm.

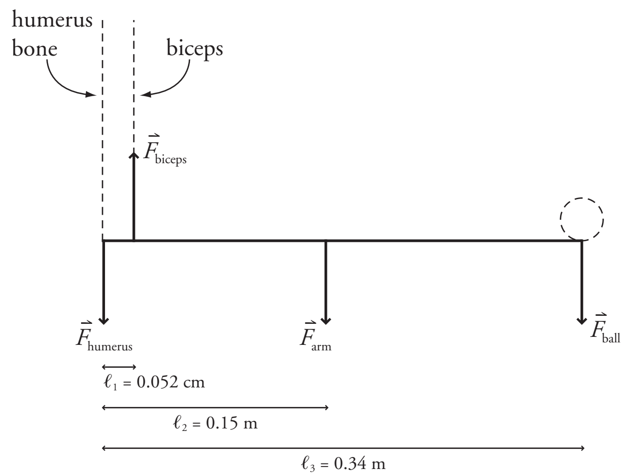
Identify the Goal

The force, F_{biceps}, the biceps muscle exerts on your forearm to hold it in place.

Variables and Constants

Identify the Variables

Known	Implied	Unknown
m_{arm} = 2.3 kg	g = 9.81 m/s^2	F_{biceps}
m_{ball} = 16 kg		
l_1 = 0.052 m		
l_2 = 0.15 m		
l_3 = 0.34 m		



Develop a Strategy

Begin by writing the equation for rotational equilibrium, using the elbow joint as the axis.

Rearrange and isolate the force exerted by the biceps.

Note that the lever arm for the force exerted by the humerus bone is zero, so this force drops out of the calculation.

Substitute numerical values and solve.

The force exerted by the biceps muscle on the forearm is 1.1×10^3 N.

Calculations

$$\Sigma\tau = 0$$

$$-l_0F_{\text{humerus}} + l_1F_{\text{biceps}} - l_2F_2 - l_3F_{\text{ball}} = 0$$

$$F_{\text{biceps}} = \frac{0 + l_2F_2 + l_3F_{\text{ball}}}{l_1}$$

$$F_{\text{biceps}} = \frac{(0.15\text{ m})(2.3\text{ kg})(9.81\text{ m/s}^2) + (0.34\text{ m})(16\text{ kg})(9.81\text{ m/s}^2)}{0.052\text{ m}}$$

$$F_{\text{biceps}} = 1091.4\text{ N}$$

$$F_{\text{biceps}} \cong 1.1 \times 10^3\text{ N}$$

Validate the Solution

The units are newtons, as required. The force is required to be large because its lever arm is small, so the answer is reasonable.

33. Frame the Problem

- Sketch the situation, indicating the directions of all the forces.
- Let the contact of the beam with the pole be the axis of rotation.
- Use the equation for *rotational equilibrium* to find the tension in the cable.
- Use the equations for *translational equilibrium* in the *x* and *y* directions to find the *x*- and *y*- components of the force of the pole on the beam.

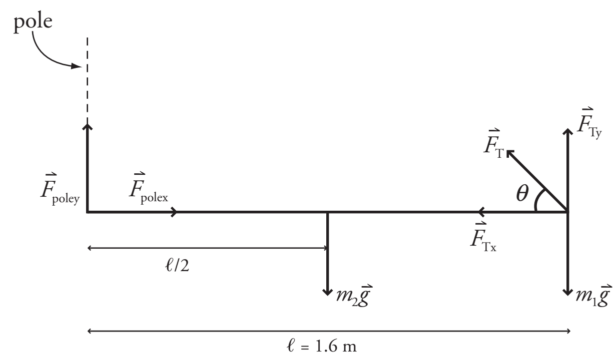
Identify the Goal

(a) The tension, $\vec{F}_{\text{T}}$, in the cable.

(b) The force exerted on the beam by the pole, $\vec{F}_{\text{pole}}$.

Variables and Constants
Identify the Variables

Known	Implied	Unknown
$m_1 = 24$ kg	$g = 9.81$ m/s^2	$\vec{F}_T$
$m_2 = 3.5$ kg		$\vec{F}_{pole}$
$l = 1.6$ m		



Develop a Strategy

Calculations

Let the contact of the beam with the pole be the axis.

$$\Sigma\tau = 0$$

Begin by writing the equation for rotational equilibrium.

$$F_T \sin\theta l - m_1 g l - m_2 g\left(\frac{l}{2}\right) = 0$$

Rearrange and isolate the tension force.

$$F_T = \frac{m_1 g - \left(\frac{m_2 g}{2}\right)}{\sin\theta}$$

Note that the length of the beam cancels.

Substitute numerical values and solve.

$$F_T = \frac{(24\text{ kg})(9.81\text{ m/s}^2) - \left(\frac{(3.5\text{ kg})(9.81\text{ m/s}^2)}{2}\right)}{\sin 33°}$$

$$F_T = 463.81\text{ N}$$

$$F_T \cong 4.6 \times 10^2\text{ N}$$

(a) The tension in the cable is 4.6×10^2 N.

Apply the equations of translational equilibrium to find the force the pole exerts on the beam.

First for the x-components.

$$\Sigma F_x = 0$$

$$F_{polex} - F_{Tx} = 0$$

$$F_{polex} = F_T \cos\theta$$

$$F_{polex} = (463.81\text{ N})\cos 33°$$

$$F_{polex} = 388.98\text{ N}$$

Then for the y-components.

$$\Sigma F_y = 0$$

$$F_{poley} + F_{Ty} - m_1 g - m_2 g = 0$$

$$F_{poley} = -F_T \sin\theta + m_1 g + m_2 g$$

$$F_{poley} = -(463.81\text{ N})\sin 33° + (24\text{ kg})(9.81\text{ m/s}^2) + (3.5\text{ kg})(9.81\text{ m/s}^2)$$

$$F_{poley} = 17.166\text{ N}$$

Use the Pythagorean theorem to find the magnitude of the force exerted by the pole.

$$\left|\vec{F}_{\text{pole}}\right|^2 = F_{\text{polex}}^2 + F_{\text{poley}}^2$$
$$\left|\vec{F}_{\text{pole}}\right|^2 = (388.98\ \text{N})^2 + (17.166\ \text{N})^2$$
$$\left|\vec{F}_{\text{pole}}\right|^2 = 151600.1\ \text{N}^2$$
$$\left|\vec{F}_{\text{pole}}\right| = 389.36\ \text{N}$$
$$\left|\vec{F}_{\text{pole}}\right| \cong 3.9\times10^2\ \text{N}$$

Use the tangent function to find the angle that the force of the pole makes with the beam.

$$\tan\theta = \frac{F_{\text{poley}}}{F_{\text{polex}}}$$
$$\theta = \tan^{-1}\frac{17.166\ \text{N}}{388.98\ \text{N}}$$
$$\theta = \tan^{-1}0.04413$$
$$\theta = 2.53^\circ$$
$$\theta \cong 2.5^\circ$$

(b) The pole exerts a force on the beam of 3.9×10^2 N at an angle of 2.5° up from the beam.

Validate the Solution

The tension in the cable is greater than the combined weights of the light and the beam (27.5 kg × 9.81 m/s² = 270 N), as it should be. The units give newtons as required. Based on the calculated tension in the cable, the value for the force exerted by the pole seems reasonable.

34. Frame the Problem

- Sketch the situation, indicating the directions of all the forces.
- Let the contact of the ladder with the ground be the axis of rotation.
- Use the equation for *rotational equilibrium* to find the normal force that the house exerts on the ladder. Note: it is assumed that the house is smooth, so at the point of contact between the ladder and the house, there is only the normal force.
- Use the equations for *translational equilibrium* in the *x* and *y* directions to find normal force at the ground and the friction force.
- Note that a common error in this problem is to get the direction of the force of friction wrong. Because the ladder wants to slip along the ground away from the house, the force of friction acts towards the house.

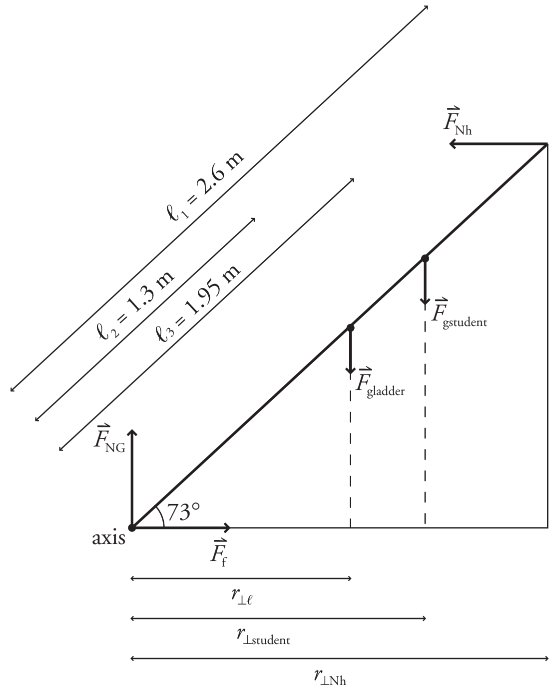
Identify the Goal

The friction force, $\vec{F}_{\text{f}}$, of the ground on the base of the ladder to prevent it from slipping.

Variables and Constants

Identify the Variables

Known	Implied	Unknown
m_1 = 55 kg	g = 9.81 m/s2	$\vec{F}_{\text{f}}$
m_2 = 7.5 kg	l_3 = 0.75 × 2.6 m	$\vec{F}_{\text{Nh}}$
l_1 = 2.6 m	= 1.95 m	$\vec{F}_{\text{NG}}$
l_2 = 1.3 m		



Develop a Strategy | **Calculations**

Begin by writing the equation for rotational equilibrium.

$$\Sigma\tau = 0$$

$$\tau_{\text{NG}} + \tau_{\text{f}} + \tau_{\text{gladder}} + \tau_{\text{gstudent}} + \tau_{\text{Nh}} = 0$$

$$r_\perp F_{\text{NG}} + r_\perp F_{\text{f}} - r_\perp F_{\text{gladder}} - r_\perp F_{\text{gstudent}} + r_\perp F_{\text{Nh}} = 0$$

Rearrange and isolate the tension force.

$$r_\perp F_{\text{Nh}} = -r_\perp F_{\text{NG}} - r_\perp F_{\text{f}} + r_\perp F_{\text{gladder}} + r_\perp F_{\text{gstudent}}$$

$$F_{\text{Nh}} = \frac{-r_\perp F_{\text{NG}} - r_\perp F_{\text{f}} + r_\perp F_{\text{gladder}} + r_\perp F_{\text{gstudent}}}{r_{\perp\text{Nh}}}$$

Note that the length of the beam cancels.

$$F_{\text{Nh}} = \frac{-(0\text{ m})F_{\text{NG}} - (0\text{ m})F_{\text{f}} + r_\perp F_{\text{gladder}} + r_\perp F_{\text{gstudent}}}{r_{\perp\text{Nh}}}$$

$$F_{\text{Nh}} = \frac{r_\perp F_{\text{gladder}} + r_\perp F_{\text{gstudent}}}{r_{\perp\text{Nh}}}$$

Consider the counterclockwise direction as positive.

Substitute numerical values and solve.

$$F_{\text{Nh}} = \frac{(1.3\text{ m})\cos 73°(7.5\text{ kg})(9.81\text{ m/s}^2) + (1.95\text{ m})\cos 73°(55\text{ kg})(9.81\text{ m/s}^2)}{(2.6\text{ m})\sin 73°}$$

$$F_{\text{Nh}} = 134.965\text{ N}$$

Apply the equations of translational equilibrium to find the force of friction.

First for the x-components.

$$\Sigma F_x = 0$$

$$F_{\text{f}} - F_{\text{Nh}} = 0$$

$$F_{\text{f}} = F_{\text{Nh}}$$

$$F_{\text{f}} = 134.965\text{ N}$$

$$F_{\text{f}} \cong 1.3 \times 10^2\text{ N}$$

The force of friction must be 1.3×10^2 N to prevent the ladder from slipping.

Note the problem is now solved and we don't need the *y*-components of the forces.

Validate the Solution
The units worked out to be newtons in each case. The value for the friction seems reasonable.

Practice Problem Solutions

Student Textbook page 509

35. Conceptualize the Problem

- Sketch the vectors representing the momentum of the two billiard balls immediately before and just after the collision. It is always helpful to superimpose an $x-y$-coordinate system on the vectors so that the origin is at the point of contact of the two balls. For calculations, use the angles that the vectors make with the *x*-axis.
- Momentum is *conserved* in the *x* and *y* directions *independently*.
- The *total momentum* of the system (ball A and B) *before* the collision is carried by *ball A* and is all in the positive *y* direction.
- *After* the collision, both balls have *momentum* in both the *y* direction and the *x* direction.
- Since the *momentum* in the *x direction* was *zero* before the collision, it must be *zero* after the collision. Therefore, the *x-components of the momentum* of the two balls after the collision must be *equal in magnitude* and *opposite in direction.*
- The *sum* of the *y*-components of the two balls *after* the collision must equal the *momentum* of billiard ball A *before* the collision.
- As components of vectors are being considered, the calculations can be done in terms of magnitudes (instead of vectors).

Identify the Goal
The velocity, $\vec{v}'_B$, of ball B after the collision

Known		Unknown
$m_A = 0.150$ kg	$\vec{v}'_A = 7.7$ m/s[40° clockwise from +*y*-direction] or 7.7 m/s [50° counter-clockwise from the +*x*-direction]	$\vec{v}'_B$
$m_B = 0.150$ kg		
$\vec{v}_A = 10.0$ m/s[+*y*-direction]		
$\vec{v}_B = 0$ m/s		

Develop a Strategy	Calculations
Write the expression for the conservation of momentum in the *x*-direction.	$m_A v_{Ax} + m_B v_{Bx} = m_A v'_{Ax} + m_B v'_{Bx}$
Solve for the velocity of ball B after the collision.	$v'_{Bx} = \dfrac{m_A v_{Ax} + m_B v_{Bx} - m_A v'_{Ax}}{m_B}$
The *x*-component of both balls before the collision was zero.	$v'_{Bx} = \dfrac{0 + 0 - (0.150\text{ kg})(7.7 \cos 50°\text{ m/s})}{(0.150\text{kg})}$
Substitute values and solve.	$v'_{Bx} = -4.9495$ m/s

Carry out the same procedure for the *y*-components.

$$m_A v_{Ay} + m_B v_{By} = m_A v'_{Ay} + m_B v'_{By}$$

$$v'_{By} = \frac{m_A v_{Ay} + m_B v_{By} - m_A v'_{Ay}}{m_B}$$

$$v'_{By} = \frac{(0.150 \text{ kg})(10.0 \text{ m/s}) + 0 - (0.150 \text{ kg})(7.7 \sin 50^\circ \text{ m/s})}{(0.150 \text{ kg})}$$

$$v'_{By} = 4.1014 \text{ m/s}$$

Use the Pythagorean theorem to find the magnitude of the resultant velocity vector of ball B.

$$\left|\vec{v}'_B\right|^2 = v'^2_{Bx} + v'^2_{By}$$

$$\left|\vec{v}'_B\right|^2 = \left(-4.9495 \ \frac{\text{m}}{\text{s}}\right)^2 + \left(4.1014 \ \frac{\text{m}}{\text{s}}\right)^2$$

$$\left|\vec{v}'_B\right|^2 = 41.3190 \frac{\text{m}^2}{\text{s}^2}$$

$$\left|\vec{v}'_B\right| = 6.4280 \text{ m/s}$$

$$\left|\vec{v}'_B\right| \cong 6.4 \text{ m/s}$$

Use the tangent function to find the direction of the velocity vector.

$$\tan\theta = \frac{v'_{By}}{v'_{Bx}}$$

$$\tan\theta = \frac{4.1014 \text{ m/s}}{-4.9495 \text{ m/s}}$$

$$\theta = \tan^{-1}(-0.82865)$$

$$\theta = -39.65^\circ$$

$$\theta \cong -40.0^\circ$$

After the collision, ball B moves with a speed of 6.4 m/s in a direction 40.0° counter-clockwise from the +*y*-direction.

Validate the Solution

Since all of the momentum before the collision was in the positive *y* direction, the *y*-component of momentum after the collision had to be in the positive *y* direction, which it was. Since there was no momentum in the *x* direction before the collision, the *x*-components of the momentum after the collision had to be in opposite directions, which they were.

36. Conceptualize the Problem

- Sketch the vectors representing the momentum of the bowling ball and pin immediately before and just after the collision. It is always helpful to superimpose an *x*–*y*-coordinate system on the vectors so that the origin is at the point of contact of the two objects. For calculations, use the angles that the vectors make with the *x*-axis.
- Momentum is *conserved* in the *x* and *y* directions *independently*.
- The *total momentum* of the system (bowling ball A and pin B) *before* the collision is carried by the *bowling ball (A)* and is all in the positive *y* direction.
- *After* the collision, both the ball and pin have *momentum* in both the *y* direction and the *x* direction.
- Since the *momentum* in the *x direction* was *zero* before the collision, it must be *zero* after the collision. Therefore, the *x-components of the momentum* of the ball and pin after the collision must be *equal in magnitude* and *opposite in direction*.
- The *sum* of the *y*-components of the ball and pin *after* the collision must equal the *momentum* of bowling ball A *before* the collision.
- As components of vectors are being considered, the calculations can be done in terms of magnitudes (instead of vectors).

Identify the Goal
The velocity, $\vec{v}'_B$, of the bowling ball A after the collision

Known

$m_A = 6.00$ kg
$m_B = 0.220$ kg
$\vec{v}_A = 1.20$ m/s[+y-direction]
$\vec{v}_B = 0$ m/s

$\vec{v}'_B = 3.60$ m/s[70.0° counter-clockwise from +y-direction]
or, 3.60 m/s [20.0° clockwise from the –x-direction]

Unknown

$\vec{v}'_A$

Develop a Strategy | **Calculations**

Write the expression for the conservation of momentum in the x-direction.

$$m_A v_{Ax} + m_B v_{Bx} = m_A v'_{Ax} + m_B v'_{Bx}$$

Solve for the velocity of the bowling ball B after the collision.

$$v'_{Ax} = \frac{m_A v_{Ax} + m_B v_{Bx} - m_A v'_{Ax}}{m_B}$$

The x-component of the system before the collision was zero. After the collision, the pin travels in the negative x-direction.

$$v'_{Ax} = \frac{0 + 0 - (0.220\text{ kg})(-3.60\cos 20^\circ\text{ m/s})}{(6.00\text{ kg})}$$

Substitute values and solve.

$$v'_{Ax} = +0.1240\text{ m/s}$$

Carry out the same procedure for the y-components.

$$m_A v_{Ay} + m_B v_{By} = m_A v'_{Ay} + m_B v'_{By}$$

$$v'_{Ay} = \frac{m_A v_{Ay} + m_B v_{By} - m_A v'_{Ay}}{m_B}$$

$$v'_{Ay} = \frac{(6.00\text{ kg})(1.20\text{ m/s}) + 0 - (0.220\text{ kg})(3.60\sin 20^\circ\text{ m/s})}{(6.00\text{ kg})}$$

$$v'_{Ay} = 1.1548\text{ m/s}$$

Use the Pythagorean theorem to find the magnitude of the resultant velocity vector of the bowling ball (A).

$$\left|\vec{v}'_A\right|^2 = v'^2_{Ax} + v'^2_{Ay}$$

$$\left|\vec{v}'_A\right|^2 = \left(0.1240\ \tfrac{\text{m}}{\text{s}}\right)^2 + \left(1.1548\ \tfrac{\text{m}}{\text{s}}\right)^2$$

$$\left|\vec{v}'_A\right|^2 = 1.3491\frac{\text{m}^2}{\text{s}^2}$$

$$\left|\vec{v}'_A\right| = 1.1615\text{ m/s}$$

$$\left|\vec{v}'_A\right| \cong 1.16\text{ m/s}$$

Use the tangent function to find the direction of the velocity vector.

$$\tan\theta = \frac{v'_{Ay}}{v'_{Ax}}$$

$$\tan\theta = \frac{1.1548\text{ m/s}}{0.1240\text{ m/s}}$$

$$\theta = \tan^{-1}(9.3129)$$

$$\theta = 83.87^\circ$$

$$\theta \cong 83.9^\circ$$

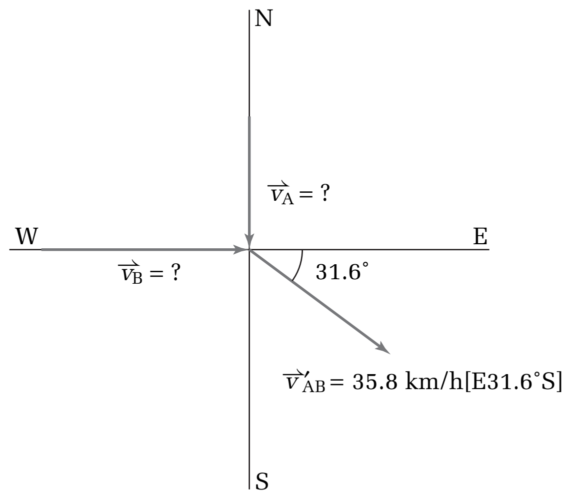
After the collision, the bowling ball (A) moves with a speed of 1.16 m/s in a direction 83.9° clockwise from the +x-axis.

Validate the Solution
The bowling ball has a much greater mass than the pin so it is expected that its speed after the collision (1.16 m/s) will only be slightly less than before the collision (1.20 m/s), as is observed. Also, the path of the bowling ball is not expected

to change much due to the collision. After the collision, the bowling ball travels only 90.0 – 83.9 = 6.1˚ from its original direction — a reasonable result.

37. Conceptualize the Problem

- Sketch a vector diagram of the momentum before and after the collision.
- Consider the two cars to be a "system." Before the collision, the *south component* of the *momentum* of the system was carried by car A and the *east component* was carried by car B.
- Momentum is conserved in the north-south direction and in the east-west direction independently.
- After the collision, the cars form *one mass* with all of the *momentum*.
- As components of vectors are being considered, the calculations can be done in terms of magnitudes (instead of vectors).



Identify the Goal

The velocities, $\vec{v}'_A$ and $\vec{v}'_B$, of the two cars before the collision

Identify the Variables

Known		Unknown
$m_A = 1750$ kg	$\vec{v}'_{AB} = 35.8\ \frac{\text{km}}{\text{h}}$[E31.6 S]	$\vec{v}_A$
$m_B = 1450$ kg		$\vec{v}_B$

Develop a Strategy	Calculations
Write the equation for conservation of momentum.	$m_A\vec{v}_A + m_B\vec{v}_B = m_A\vec{v}'_A + m_B\vec{v}'_B$
Work with the north-south direction only. Modify the equation to show that car B was moving directly east before the crash; its north-south momentum was zero. After the crash, the cars were combined.	$m_A v_A[\text{S}] = (m_A + m_B)v'_{AB}[\text{S}]$ (Note that vector notations are not included, because you are considering only the north-south component of the velocities.)
Solve the equation for the original velocity of car A.	$v_A[\text{S}] = \frac{(m_A + m_B)v'_{AB}[\text{S}]}{m_A}$

Substitute the values and solve.

$$v_A[S] = \frac{(1750\text{ kg} + 1450\text{ kg})35.8\ \frac{\text{km}}{\text{h}}\sin 31.6°}{1750\text{ kg}}$$

$$v_A[S] = \frac{(3200\text{ kg})(35.8\ \frac{\text{km}}{\text{h}}\sin 0.52398)}{1750\text{ kg}}$$

$$v_A[S] = 34.301\ \frac{\text{km}}{\text{h}}$$

$$v_A \cong 34.3\ \frac{\text{km}}{\text{h}}[S]$$

(Note that the south component of car A's velocity before the crash was most of the total velocity.)

Carry out the same procedure for the east-west direction of the momentum.

$$m_B v_B[E] = (m_A + m_B)v'_{AB}[E]$$

$$v_B[E] = \frac{(m_A + m_B)v'_{AB}[E]}{m_B}$$

$$v_B[E] = \frac{(1750\text{ kg} + 1450\text{ kg})(35.8\ \frac{\text{km}}{\text{h}}\cos 31.6°)}{1450\text{ kg}}$$

$$v_B[E] = 67.292\ \frac{\text{km}}{\text{h}}$$

$$v_B \cong 67.3\ \frac{\text{km}}{\text{h}}[E]$$

Car A was travelling 34.3 km/h south and car B was travelling 67.3 km/h east at the instant before the crash.

Validate the Solution

The units all cancelled to give km/h, which is correct for velocity. After the collision, the cars travel together at an angle of less than 45°, that is, more towards the east than towards the south [E31.6°S]. Therefore, it is expected that the momentum of car B, which is travelling east before the collision, will be greater than the momentum of car A. Before the collision, car A's momentum is $m_A v_A[S] = (1750\text{ kg})(34.3\text{ km/h}) = 6.0 \times 10^4\text{ kg km/h}[S]$, and car B's momentum is $m_B v_B[E] = (1450\text{ kg})(67.3\text{ km/h}) = 9.8 \times 10^4\text{ kg km/h}[E]$, so, the eastward momentum before the collision is greater than the southward momentum, as expected.

Practice Problem Solutions

Student Textbook page 513

38. Frame the Problem

- Make a sketch of the momentum vectors after the explosion.
- After an explosion, the *vector sum* of the *momentum* of all the fragments must be zero.

Identify the Goal

The mass, m_3, and velocity, $\vec{v}_3$, of the third fragment after the explosion.

Identify the Variables

Known	Unknown
$m_1 = 1.3$ kg	m_3
$m_2 = 1.2$ kg	$\vec{v}_3$
$\vec{v}_1 = 1.8$ m/s	
$\theta_1 = 52°$ ccw from + x - axis	
$\vec{v}_2 = 1.8$ m/s	
$\theta_2 = 61°$ cw from − x - axis	

Develop a Strategy / Calculations

Find the *x*- and *y*-components of the momentum of fragments 1 and 2.

$$p_{1x} = m_1 v_{1x} \qquad p_{2x} = m_2 v_{2x}$$
$$p_{1x} = m_1 |\vec{v}_1| \cos 52° \qquad p_{2x} = -m_2 |\vec{v}_2| \cos 61°$$

Because fragment 2 is in the 3rd quadrant, both its components are negative.

$$p_{1y} = m_1 v_{1y} \qquad p_{2y} = m_2 v_{2y}$$
$$p_{1y} = m_1 |\vec{v}_1| \sin 52° \qquad p_{2y} = m_2 |\vec{v}_2| \sin 61°$$

Sum the *x*- and *y*- components of the momentum individually.

Begin with the *x*-components.

$$p_{1x} + p_{2x} + p_{3x} = 0$$
$$m_1 |\vec{v}_1| \cos 52° - m_2 |\vec{v}_2| \cos 61° + p_{3x} = 0$$
$$p_{3x} = m_2 |\vec{v}_2| \cos 61° - m_1 |\vec{v}_1| \cos 52°$$
$$p_{3x} = (1.2 \text{ kg})(2.5 \text{ m/s})\cos 61° - (1.3 \text{ kg})(1.8 \text{ m/s})\cos 52°$$
$$p_{3x} = 0.01378 \text{ kg} \cdot \text{m/s}$$

Then repeat for the *y*-components.

$$p_{1y} + p_{2y} + p_{3y} = 0$$
$$m_1 |\vec{v}_1| \sin 52° + m_2 |\vec{v}_2| \sin 61° + p_{3y} = 0$$
$$p_{3y} = -m_2 |\vec{v}_2| \sin 61° - m_1 |\vec{v}_1| \sin 52°$$
$$p_{3y} = -(1.2 \text{ kg})(2.5 \text{ m/s})\sin 61° - (1.3 \text{ kg})(1.8 \text{ m/s})\sin 52°$$
$$p_{3y} = -4.4678 \text{ kg} \cdot \text{m/s}$$

Use the Pythagorean theorem to find the magnitude of the momentum of the third fragment.

$$|\vec{p}_3|^2 = p_{3x}^2 + p_{3y}^2$$
$$|\vec{p}_3|^2 = (0.01378 \text{ kg m/s})^2 + (4.4678 \text{ kg m/s})^2$$
$$|\vec{p}_3|^2 = 19.9614 \text{ (kg m/s)}^2$$
$$|\vec{p}_3| = 4.4678 \text{ kg} \cdot \text{m/s}$$
$$|\vec{p}_3| \cong 4.5 \text{ kg} \cdot \text{m/s}$$

Use the tangent function to find the direction of the momentum of fragment 3.

$$\tan\theta = \frac{p_{3y}}{p_{3x}}$$
$$\theta = \tan^{-1} \frac{4.4678 \text{ kg m/s}}{0.01378 \text{ kg m/s}}$$
$$\theta = \tan^{-1} 324.22$$
$$\theta = 89.822°$$
$$\theta \cong 89.8°$$

Find the mass of the third fragment from the total mass.

$$m_1 + m_2 + m_3 = 3.5 \text{ kg}$$
$$m_3 = 3.5 \text{ kg} - 1.3 \text{ kg} - 1.2 \text{ kg}$$
$$m_3 = 1.0 \text{ kg}$$

Find the velocity of fragment 3 from the definition of momentum.

$$|\vec{p}| = m|\vec{v}|$$

$$|\vec{v}| = \frac{|\vec{p}|}{m}$$

$$|\vec{v}| = \frac{4.5 \text{ kg m/s}}{1.0 \text{ kg}}$$

$$|\vec{v}| = 4.5 \text{ m/s}$$

The velocity of the third fragment was 4.5 m/s at an angle of 89.8° ccw from the +*x*-axis.

Validate the Solution

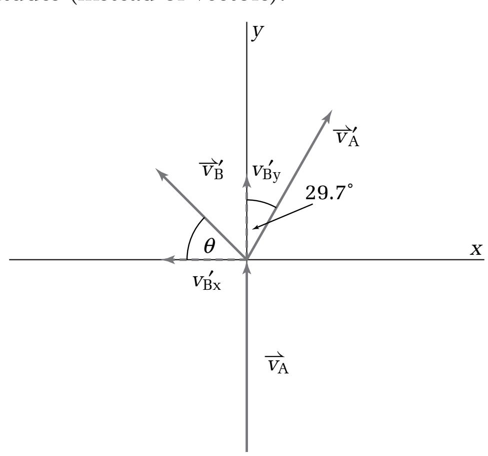
The units worked properly in each case. The third fragment has the smallest mass of the three fragments, so it is expected to have a large velocity, and it does. The value seems reasonable.

Practice Problem Solutions

Student Textbook page 515

39. Conceptualize the Problem

- Sketch the vectors representing the momentum of the two billiard balls immediately before and just after the collision. Superimpose an *x*–*y*-coordinate system on the vectors so that the origin is at the point of contact of the two balls. For calculations, use the angles that the vectors make with the *x*-axis.
- Momentum is *conserved* in the *x* and *y* directions *independently*.
- The *total momentum* of the system (ball A and B) *before* the collision is carried by *ball A* and is all in the positive *y* direction.
- *After* the collision, both balls have *momentum* in both the *y* direction and the *x* direction.
- Since the *momentum* in the *x* direction was *zero* before the collision, it must be *zero* after the collision. Therefore, the *x-components of the momentum* of the two balls after the collision must be *equal in magnitude* and *opposite in direction*.
- The *sum* of the *y*-components of the two balls *after* the collision must equal the *momentum* of billiard ball A *before* the collision.
- *Total momentum* is always conserved in a collision.
- If the collision is *elastic*, total kinetic energy must also be conserved.
- As components of vectors are being considered, the calculations can be done in terms of magnitudes (instead of vectors).



Identify the Goal

Whether the total kinetic energy of the system before the collision, E_{kA}, is equal to the total kinetic energy of the system after the collision, $E'_{kA} + E'_{kB}$

Identify the Variables

Known		Unknown
$m_A = 0.155$ kg	$\vec{v}'_A = 9.56$ m/s[29.7°	$\vec{v}'_B$
$m_B = 0.155$ kg	clockwise from +y-direction]	E_{kA}
$E_{kB} = 0$ J	= 9.56 m/s[60.3° counter-	E'_{kA} E'_{kB}
$\vec{v}_A = 12.5$ m/s[+y-direction]	clock-wise from +x-direction]	
$\vec{v}_B = 0$ m/s		

Develop a Strategy **Calculations**

Write the expression for the conservation of momentum in the x-direction.

$$m_A v_{Ax} + m_B v_{Bx} = m_A v'_{Ax} + m_B v'_{Bx}$$

Solve for the velocity of ball B after the collision.

$$v'_{Bx} = \frac{m_A v_{Ax} + m_B v_{Bx} - m_A v'_{Ax}}{m_B}$$

The x-component of both balls before the collision was zero. Notice the masses divide out.

$$v'_{Bx} = \frac{0 + 0 - (0.155\text{ kg})(9.56\cos 60.3°\text{ m/s})}{(0.155\text{kg})}$$

Substitute values and solve.

$$v'_{Bx} = -4.7366\text{ m/s}$$

Carry out the same procedure for the y-components. Notice the masses divide out.

$$m_A v_{Ay} + m_B v_{By} = m_A v'_{Ay} + m_B v'_{By}$$

$$v'_{By} = \frac{m_A v_{Ay} + m_B v_{By} - m_A v'_{Ay}}{m_B}$$

$$v'_{By} = \frac{(0.155\text{ kg})(12.5\text{ m/s}) + 0 - (0.155\text{ kg})(9.56\sin 60.3°\text{ m/s})}{(0.155\text{ kg})}$$

$$v'_{By} = 4.1959\text{ m/s}$$

Use the Pythagorean theorem to find the magnitude of the resultant velocity vector of ball B.

$$|\vec{v}'_B|^2 = v'^2_{Bx} + v'^2_{By}$$

$$|\vec{v}'_B|^2 = \left(-4.7366\ \tfrac{\text{m}}{\text{s}}\right)^2 + \left(4.1959\ \tfrac{\text{m}}{\text{s}}\right)^2$$

$$|\vec{v}'_B|^2 = 40.0410\ \tfrac{\text{m}^2}{\text{s}^2}$$

$$|\vec{v}'_B| = 6.3278\text{ m/s}$$

$$|\vec{v}'_B| \cong 6.3\text{ m/s}$$

Use the tangent function to find the direction of the velocity vector.

$$\tan\theta = \frac{v'_{By}}{v'_{Bx}}$$

$$\tan\theta = \frac{4.1959\text{ m/s}}{-4.7366\text{ m/s}}$$

$$\theta = \tan^{-1}(-0.88585)$$

$$\theta = -41.536°$$

$$\theta \cong -41.5°$$

After the collision, ball B moves with a speed of 6.3 m/s in a direction 41.5° clockwise from the –x-direction.

Calculate the kinetic energy of ball A before the collision.

$$E_{kA} = \frac{1}{2} m_A v_A^2$$

$$E_{kA} = \frac{1}{2}(0.155\ \text{kg})\left(12.5\ \frac{\text{m}}{\text{s}}\right)^2$$

$$E_{kA} = 12.1\ \text{J}$$

Calculate the sum of the kinetic energies of the balls after the collision.

$$E'_k = \frac{1}{2} m_A v'^2_A + \frac{1}{2} m_B v'^2_B$$

$$E'_k = \frac{1}{2}(0.155\ \text{kg})\left(9.56\ \frac{\text{m}}{\text{s}}\right)^2 + \frac{1}{2}(0.155\ \text{kg})\left(6.33\ \frac{\text{m}}{\text{s}}\right)^2$$

$$E'_k = 10.2\ \text{J}$$

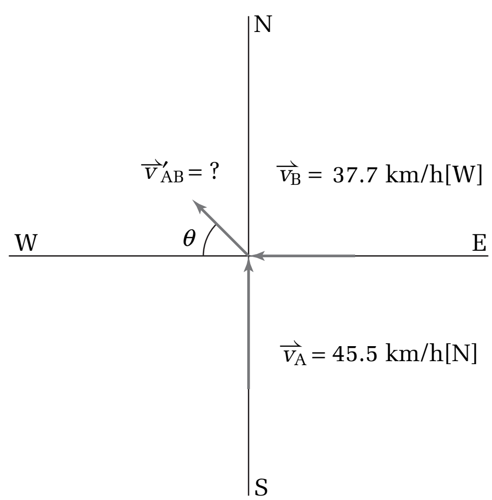
The kinetic energy before the collision is greater than the kinetic energy after the collision by almost 2.0 J, so the collision was not elastic.

Validate the Solution

After the collision, the stationary billiard ball travels away at a larger angle (41.5° to the left) than the moving billiard ball (29.7° to the right). Because it is a "glancing" collision, it is reasonable that energy could be dissipated during the collision so that the kinetic energy after the collision is less than the kinetic energy before.

40. Conceptualize the Problem

- Sketch the vectors representing the momentum of the two cars immediately before and just after the collision.
- Consider the two cars to be a "system." Before the collision, the *north component* of the *momentum* of the system was carried by car A and the *west component* was carried by car B.
- Momentum is conserved in the north-south direction and in the east-west direction independently.
- After the collision, the cars form *one mass* with all of the *momentum*.
- *Total momentum* is always conserved in a collision.
- If the collision is *elastic*, total kinetic energy must also be conserved.



Identify the Goal

The total momentum of the system, $\vec{p}'_{AB}$, after the collision

Whether the total kinetic energy of the system before the collision, $E_{kA} + E_{kB}$, is equal to the total kinetic energy of the system after the collision, $E'_{kA} + E'_{kB}$

Identify the Variables

Known

$m_A = 1735$ kg $\vec{v}_A = 45.5$ km/h[N]

$m_B = 2540$ kg $\vec{v}_B = 37.7$ km/h[W]

Unknown

$\vec{v}'_{AB}$ $\vec{P}'_{AB}$

E_{kA} E'_{kB}

E'_{kA} E'_{kB}

Develop a Strategy	Calculations
Write the expression for the conservation of momentum in the N–S–direction.	$m_A v_A[N] + m_B v_B[N] = (m_A + m_B)v'_{AB}[N] \sin\theta$
Notice θ is the angle measured clockwise from the negative x-axis (the west direction).	$m_A v_A[N] = (m_A + m_B)v'_{AB}[S] \sin\theta$ (1)

The initial momentum of car B in this direction is zero.

Write the expression for the conservation of momentum in the E–W–direction.	$m_A v_A[W] + m_B v_B[W] = (m_A + m_B)v'_{AB}[W] \cos\theta$
The initial momentum of car A in this direction is zero.	$m_B v_B[W] = (m_A + m_B)v'_{AB}[W] \cos\theta$ (2)
There are now two equations with two unknowns, the final combined velocity of the cars, $\vec{v}'_{AB}$, and the direction they travel, θ.	$\dfrac{m_A v_A[N]}{m_B v_B[W]} = \dfrac{(m_A + m_B)v'_{AB}[N] \sin\theta}{(m_A + m_B)v'_{AB}[W] \cos\theta}$
Divide equation (1) by equation (2) and solve for the angle.	$\dfrac{m_A v_A}{m_B v_B} = \dfrac{\sin\theta}{\cos\theta} = \tan\theta$
The directions, combined masses and velocity cancels from the equation. The trigonometry terms simplify.	$\tan\theta = \dfrac{(1735\text{ kg})(45.5\text{ km/h})}{(2540\text{ kg})(37.7\text{ km/h})}$ $\theta = \tan^{-1}(0.82440)$ $\theta = 39.502°$ $\theta \cong 39.5°$
Substitute known values and solve. The cars move together after the collision in a direction W39.5°N. Use this result in equation (1).	$m_A v_A[N] = (m_A + m_B)v'_{AB}[N] \sin\theta$ $v'_{AB} = \dfrac{m_A v_A}{(m_A + v_B)\sin\theta}$
Substitute known values and solve.	$v'_{AB} = \dfrac{(1735\text{ kg})(45.5\text{ km/h})}{(1735\text{ kg} + 2540\text{ kg})\sin(39.502°)}$ $v'_{AB} = 29.03$ km/h
Determine the momentum of the combined cars immediately after the collision.	$\vec{p}'_{AB} = m_{AB}\vec{v}'_{AB}$ $\vec{p}'_{AB} = (1735\text{ kg} + 2540\text{ kg})(29.03\text{ km/h})[W39.5°N]$ $\vec{p}'_{AB} = 1.2410 \times 10^5$ kg km/h[W39.5°N] $\vec{p}'_{AB} \cong 1.24 \times 10^5$ kg km/h[W39.5°]

The combined momentum of the cars after the collision is 1.24×10^5 kg km/h [W39.5°N]. Calculate the combined kinetic energy of the cars before the collision.

$$E_k = E_{kA} + E_{kB}$$
$$E_k = \frac{1}{2} m_A v_A^2 + \frac{1}{2} m_B v_B^2$$
$$E_k = \frac{1}{2}(1735\ \text{kg})\left(45.5\ \frac{\text{km}}{\text{h}}\right)^2 + \frac{1}{2}(2540\ \text{kg})\left(37.7\ \frac{\text{km}}{\text{h}}\right)^2$$
$$E_k = 3.60 \times 10^6\ \text{kg km}^2/\text{h}^2$$

Calculate the sum of the kinetic energies of the cars after the collision.

$$E_k' = \frac{1}{2}(m_A + m_B)v_{AB}'^2$$
$$E_k' = \frac{1}{2}(1735\ \text{kg} + 2540\ \text{kg})\left(29.03\ \frac{\text{km}}{\text{h}}\right)^2$$
$$E_k' = 1.80 \times 10^6\ \text{kg km}^2/\text{h}^2$$

The kinetic energy of the system before the collision is a factor of two greater than the kinetic energy after the system, so the collision is not elastic.

Validate the Solution
The speed of the combined cars after the collision, 29.0 km/h, is lower than the speed of either car before the collision, as expected. Also, as this is a collision between cars, it is expected that some kinetic energy will be lost to deforming the cars during the collision, and it is not expected to be elastic, as is the case.

Practice Problem Solutions

Student Textbook pages 524–525

41. Conceptualize the Problem

- Sketch the positions of the bullet and pendulum bob just before the collision, just after the collision, and with the pendulum at its highest point.
- When the bullet hit the pendulum, total *momentum* was *conserved*.
- If you can find the *velocity* of the combined bullet and pendulum bob after the collision, you can use conservation of momentum to find the *velocity* of the bullet before the collision.
- The collision was completely inelastic so total *kinetic energy* was *not* conserved.
- However, you can assume that the friction of the pendulum is negligible, so *mechanical energy* of the pendulum was *conserved*.
- The *gravitational potential energy* of the combined masses at the highest point of the pendulum is equal to the *kinetic energy* of the combined masses at the lowest point of the pendulum.
- If you know the kinetic energy of the combined masses just after the collision, you can find the *velocity* of the masses just *after* the collision.
- Use the subscripts "b" for bullet and "p" for pendulum.

Identify the Goal
The velocity, v_b, of the bullet just before it hit the ballistic pendulum

Identify the Variables

Known	Implied	Unknown
$m_b = 12.5$ g h = 9.55 cm	$g = 9.81\frac{\text{m}}{\text{s}^2}$	v_b E_g
$m_p = 2.37$ kg		v_p E_k

Develop a Strategy | **Calculations**

To find the velocity of the combined masses of the bullet and pendulum bob just after the collision, use the relationship that describes the conservation of mechanical energy of the pendulum.

$$E_{\text{k(bottom)}} = E_{\text{g(top)}}$$

Substitute the expressions for kinetic energy and gravitational potential energy known from previous physics courses. Solve for velocity. Convert all units to SI units.

$$\frac{1}{2}\cancel{m}v_{\text{bottom}}^2 = \cancel{m}g\Delta h$$
$$v_{\text{bottom}}^2 = 2g\Delta h$$
$$v_{\text{bottom}} = \sqrt{2\Delta h}$$
$$v_{\text{bottom}} = \sqrt{2\left(9.81\frac{\text{m}}{\text{s}^2}\right)(9.55\text{ cm})\left(\frac{1\text{ m}}{100\text{ cm}}\right)}$$

Define the direction of the bullet as positive during and immediately after the collision.

$$v_{\text{bottom}} = \sqrt{1.87371\frac{\text{m}^2}{\text{s}^2}}$$
$$v_{\text{bottom}} = \pm 1.3688\frac{\text{m}}{\text{s}}$$

Apply the conservation of momentum to find the velocity of the bullet before the collision. Convert all units to SI units. Motion is in a single dimension so magnitudes can be used in the calculation.

$$m_{\text{b}}v_{\text{b}} + m_{\text{p}}v_{\text{p}} = m_{\text{b}}v'_{\text{b}} + m_{\text{p}}v'_{\text{p}}$$
$$m_{\text{b}}v_{\text{b}} + 0 = (m_{\text{b}} + m_{\text{p}})v'_{\text{bp}}$$
$$v_{\text{b}} = \frac{(m_{\text{b}} + m_{\text{p}})v_{\text{bp}}}{m_{\text{b}}}$$
$$v_{\text{b}} = \frac{\left[12.5\,\cancel{\text{g}}\left(\frac{1\text{ kg}}{1000\,\cancel{\text{g}}}\right) + 2.37\text{ kg}\right]1.3688\,\frac{\text{m}}{\text{s}}}{12.5\,\cancel{\text{g}}\left(\frac{1\text{ kg}}{1000\,\cancel{\text{g}}}\right)}$$
$$v_{\text{b}} = 260.89\,\frac{\text{m}}{\text{s}}$$
$$v_{\text{b}} \cong 261\,\frac{\text{m}}{\text{s}}$$

The speed of the bullet just before the collision was about 261 m/s in the positive direction.

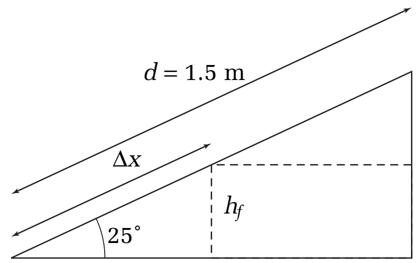
Validate the Solution

In both calculations, the units cancelled to give metres per second, which is correct for velocity. A speed of 261 m/s for a bullet is a reasonable result.

42. Conceptualize the Problem

- Sketch the positions of the ball of putty and the pendulum bob just before the collision, just after the collision, and with the cart at its highest point on the track.
- When the putty hit the cart, total *momentum* was *conserved.*
- The *velocity* of the putty before the collision is known, so conservation of momentum can be used to find the *velocity* of the putty–cart system immediately after impact.
- The collision was completely inelastic so total *kinetic energy* was *not* conserved.
- However, because the cart ascended an air track, it can be assumed that friction can be ignored, so *mechanical energy* of the putty–cart system was *conserved.*
- The *kinetic energy* of the putty–cart system at its maximum height up the track is zero.
- The *gravitational potential energy* of the combined masses at the highest point of the air track is equal to the *kinetic energy* of the combined masses at the lowest point of

the air track (that is, before the cart begins its ascent up the track, when the putty hits the cart).
- The *gravitational potential energy* gives the height up the track that the cart ascends. From this, the *distance* the putty–cart system travels up the track can be calculated from trigonometry.



Identify the Goal

The distance, Δx, the putty–cart system travels up the track (and whether it travels to the end)

Identify the Variables

Known		Implied	Unknown
$m_p = 23$ g	$v_p = 4.2$ m/s	$g = 9.81\ \frac{\text{m}}{\text{s}^2}$	v'_{pc}
$m_c = 225$ g	$d = 1.5$ m		h_f
$\theta = 25°$			Δx

Develop a Strategy / **Calculations**

Apply the conservation of momentum to find the velocity of the putty–cart system immediately after the collision. Convert all units to SI units.

$$m_p v_p + m_c v_c = m_p v'_p + m_c v'_c$$
$$m_p v_p + 0 = (m_p + m_c) v'_{pc}$$

Before the cart ascends the ramp, all the motion is in one dimension, so magnitudes can be used in the calculation.

$$v'_{pc} = \frac{m_p v_p}{(m_p + m_c)}$$

$$v'_{pc} = \frac{\left(23.5\ \cancel{\text{g}} \times \frac{1\ \text{kg}}{1000\ \cancel{\text{g}}}\right)(4.2\ \text{m/s})}{\left(23.5\ \cancel{\text{g}} \times \frac{1\ \text{kg}}{1000\ \cancel{\text{g}}} + 225\ \cancel{\text{g}} \times \frac{1\ \text{kg}}{1000\ \cancel{\text{g}}}\right)}$$

$$v'_{pc} = 0.3895\ \frac{\text{m}}{\text{s}}$$

To find the height that the combined masses travel up the track after the collision, use the relationship that describes the conservation of mechanical energy of the system.

$$E_{k(\text{bottom})} = E_{g(\text{top})}$$

The velocity of the combined putty–cart system is the velocity at the bottom of the track.

$$\frac{1}{2} \cancel{m}_{pc}\, v'^2_{pc} = \cancel{m}_{pc}\, g h_f$$

Solve for the final height that the system travels up the track.

$$h_f = \frac{\frac{1}{2} v'^2_{pc}}{g}$$

$$h_f = \frac{\frac{1}{2}(0.3895\ \text{m/s})^2}{9.81\ \text{m/s}^2}$$

$$h_f = 0.007732\ \text{m}$$

Use trigonometry to determine the distance the cart travels along the track. Let Δx be the distance along the slanted track (i.e. the hypotenuse of the right triangle). The sine function applies. Solve for Δx.

$$\sin\theta = \frac{h_f}{\Delta x}$$
$$\Delta x = \frac{0.007732\ \text{m}}{\sin 25^\circ}$$
$$\Delta x = 0.0183\ \text{m}$$
$$\Delta x \cong 0.018\ \text{m}$$

The system rolls about 0.018 m along the track and thus, it stops far short of the end.

Validate the Solution

The ball of putty has a low mass (23 g) so it is not expected to give the cart, whose mass is ten times greater, a large velocity. The velocity of the putty–cart system is only about 0.4 m/s, which seems about right. This is a low velocity, so it is not expected that the system will travel very far up the sloped track. A distance of 0.018 m = 1.8 cm seems reasonable. In the calculation of the final height, the units worked out to be metres, as required.

43. Conceptualize the Problem

- Sketch the cars just before, at the moment of, and after the collision, when they came to a stop.
- Total *momentum* is conserved during the collision. However, momentum conservation cannot be applied first because there are two unknowns (the velocity of the second car before the collision, and the combined velocity after the collision). It must be applied after the combined velocity of the two cars is found.
- Because the cars stuck together, the collision was *completely inelastic*, so total *kinetic* energy was *not* conserved. Some kinetic energy was lost to sound, heat, and deformation of the metal during the collision.
- Some *kinetic energy* remained after the collision.
- The force of *friction* did *work* on the moving cars, converting the remaining kinetic energy into heat.
- From the l*aw of conservation of energy*, the work done by the force of *friction* is equal to the *kinetic energy* of the cars at the instant after the collision.
- Since the motion is in one direction, use a plus sign to symbolize that direction.
- Let car A be the stopped car and car B be the moving car.

Identify the Goal

The speed, v_B, of the second car before the collision

Identify the Variables

Known		Implied	Unknown
$m_A = 1875$ kg	$v_A = 0$ m/s	$g = 9.81\ \frac{\text{m}}{\text{s}^2}$	v_B
$m_B = 2135$ kg			v'_{AB}
$\mu = 0.750$	$\Delta d = 4.58$ m		

Develop a Strategy / **Calculations**

Due to the law of conservation of energy, the work done on the cars by the force of friction is equal to the kinetic energy of the connected cars after the collision.

$$W_{(\text{to stop cars})} = E_{k\ (\text{immediately after collision})}$$

Substitute the expressions for work and kinetic energy into the equations.

$$F\Delta d = \frac{1}{2} m_{AB}\, v'^2_{AB}$$

Both quantities (work and kinetic energy) are scalars, so only the magnitudes of the displacement and velocity are needed.

Friction is the force doing the work, and it is always parallel to the direction of motion. Substitute the formula for the force of friction.

$$F_f\Delta d = \frac{1}{2} m_{AB}\, v'^2_{AB}$$
$$\mu F_N\Delta d = \frac{1}{2} m_{AB}\, v'^2_{AB}$$

Since the cars are moving horizontally, the normal force is the weight of the cars. Substitute the weight into the expression and solve for the velocity.

$$\mu m_{AB} g\Delta d = \frac{1}{2} m_{AB}\, v'^2_{AB}$$
$$v'^2_{AB} = 2\mu g\Delta d$$
$$v'_{AB} = \sqrt{2\mu g\Delta}$$
$$v'_{AB} = \sqrt{2(0.750)(9.81\ \text{m/s}^2)(4.58\ \text{m})}$$
$$v'_{AB} = 8.209\ \text{m/s}$$

Apply the conservation of momentum to find the velocity of the second car (car B) immediately before the collision.

$$m_A v_A + m_B v_B = m_A v'_A + m_B v'_B$$
$$0 + m_B v_B = (m_A + m_B)v'_{AB}$$

As the motion is in one dimension, use magnitudes in the calculation.

$$v_B = \frac{(m_A + m_B)v'_{AB}}{m_B}$$
$$v_B = \frac{(1875\ \text{kg} + 2135\ \text{kg})(8.209\ \text{kg})}{(2135\ \text{kg})}$$
$$v_B = 15.418\ \text{m/s}$$
$$v_B \cong 15.4\ \text{m/s}$$

The speed of the second car before the collision was about 15.4 m/s.

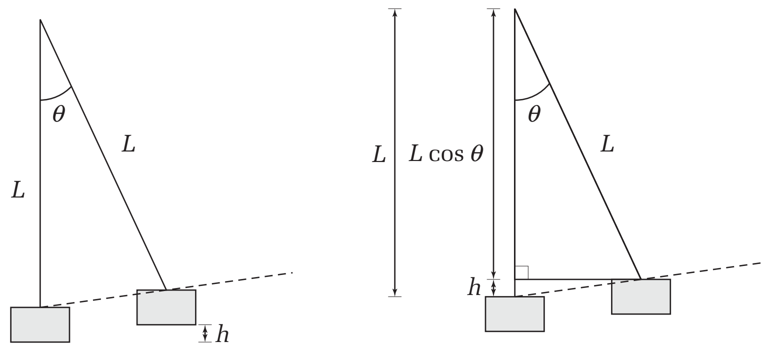
Validate the Solution

In the work-kinetic energy calculation, the units cancelled to give m/s, as required for velocity. The velocity of car B before the collision, 15.4 m/s, is about 55.5 km/h, which is a reasonable highway speed.

44. Conceptualize the Problem

- Sketch the positions of the ball and pendulum box just before the collision, just after the collision, and with the pendulum at its highest point.
- When the ball hit the pendulum, total *momentum* was *conserved*.
- If you can find the *velocity* of the combined ball and pendulum box after the collision, you can use conservation of momentum to find the *velocity* of the ball before the collision.
- The collision was completely inelastic so total *kinetic energy* was *not* conserved.
- However, you can assume that the friction of the pendulum is negligible, so *mechanical energy* of the pendulum was *conserved*.
- The *gravitational potential energy* of the combined masses at the highest point of the pendulum is equal to the kinetic energy of the combined masses at the lowest point of the pendulum.

- The *highest point* of the pendulum can be determined using the measured angle and trigonometry.
- If you know the kinetic energy of the combined masses just after the collision, you can find the *velocity* of the masses just *after* the collision.
- Use the subscripts "b" for ball and "p" for pendulum.



$h = L - L\cos\theta$

Identify the Goal

The velocity, v_{b}, of the ball just before it hit the pendulum box

Identify the Variables

Known	Implied	Unknown
$m_{\mathrm{b}} = 0.350$ kg	$g = 9.81\,\frac{\mathrm{m}}{\mathrm{s}^2}$	v_{b}
$m_{\mathrm{p}} = 5.64$ kg		v'_{bp}
$\theta = 20.0°$ $\quad L = 0.955$ m		Δh

Develop a Strategy | **Calculations**

To find the velocity of the combined masses of the ball and pendulum box just after the collision, use the relationship that describes the conservation of mechanical energy of the pendulum.

$$E_{\mathrm{k(bottom)}} = E_{\mathrm{g(top)}}$$

Substitute the expressions for kinetic energy and gravitational potential energy known from previous physics courses. Solve for velocity. The height of the box is determined from trigonometry: $\Delta h = L - L\cos\theta = L(1 - \cos\theta)$. Define the direction of the ball as positive during and immediately after the collision.

$$\frac{1}{2}mv_{\mathrm{bottom}}^2 = mg\Delta h$$

$$v_{\mathrm{bottom}}^2 = 2g\Delta h$$

$$v_{\mathrm{bottom}} = \sqrt{2g\Delta h}$$

$$v_{\mathrm{bottom}} = \sqrt{2gL(1 - \cos\theta)}$$

$$v_{\mathrm{bottom}} = \sqrt{2\left(9.81\,\frac{\mathrm{m}}{\mathrm{s}^2}\right)(0.955\ \mathrm{m})(1 - \cos 20.0°)}$$

$$v_{\mathrm{bottom}} = \pm 1.06301\,\frac{\mathrm{m}}{\mathrm{s}}$$

Apply the conservation of momentum to find the velocity of the ball before the collision.

$$m_b v_b + m_p v_p = m_b v'_b + m_p v'_p$$
$$m_b v_b + 0 = (m_b + m_p) v'_{bp}$$
$$v_b = \frac{(m_b + m_p) v'_{bp}}{m_b}$$

As the motion is in one dimension,
use magnitudes in the calculation.

$$v_b = \frac{[0.350 \text{ kg} + 5.64 \text{ kg}]1.06301 \frac{\text{m}}{\text{s}}}{0.350 \text{ kg}}$$
$$v_b = 18.1926 \frac{\text{m}}{\text{s}}$$
$$v_b \cong 18.2 \frac{\text{m}}{\text{s}}$$

The speed of the ball just before the collision was about 18.2 m/s in the positive direction.

Validate the Solution

The angle of deflection, 20.0°, is relatively small, so the velocity of the ball is not expected to rival that of a major league pitcher. A value of 18.2 m/s seems reasonable. Note that a deflection angle of greater than 52° would be required for the speed of the pitch to be greater than 46 m/s.

45. Conceptualize the Problem

- Sketch the position of the boulder when it is on the side of the mountain, just before the collision with the shack, and just after the collision.
- The *gravitational potential energy* of the boulder on the side of the mountain is equal to the *kinetic energy* of the boulder immediately before the collision.
- When the boulder hit the shack, total *momentum* was *conserved*.
- Because you can find the *velocity* of the boulder before the collision, you can use conservation of momentum to find the velocity of the boulder–shack system immediately after the collision.
- The collision was completely inelastic (the boulder and shack stuck together) so *kinetic energy* was *not* conserved. Some kinetic energy was lost to sound, heat, and deformation of the shack during the collision.
- Some *kinetic energy* remained after the collision.
- The *force of friction* did work on the boulder–shack system, converting the remaining kinetic energy to heat.
- From the *law of conservation* of *energy*, the *work* done by the *force of friction* is equal to the *kinetic energy* of the system at the instant after the collision.
- Since the motion is in one direction, use a plus sign to symbolize direction.

Identify the Goal

The velocity, v_{bs}, of the boulder–shack system at the instant they began to slide across the ice, and the distance, Δd, that they slide across the ice

Identify the Variables

Known	Implied	Unknown
$m_b = 55.6$ kg	$g = 9.81 \frac{\text{m}}{\text{s}^2}$	v_b
$m_s = 204$ kg		v'_{bs}
$\Delta h = 14.6$ m		Δd
$\mu = 0.392$		

Develop a Strategy | **Calculations**

To find the velocity of the combined masses of the boulder and shack just after the collision, use the relationship that describes the conservation of mechanical energy of the boulder.

$$E_{\text{k(bottom)}} = E_{\text{g(top)}}$$

Substitute the expressions for kinetic energy and gravitational potential energy known from previous physics courses. Solve for velocity. Define the direction of the boulder as positive during and immediately after the collision.

$$\frac{1}{2}\cancel{m}v^2_{\text{bottom}} = \cancel{m}g\Delta h$$

$$v^2_{\text{bottom}} = 2g\Delta h$$

$$v_{\text{bottom}} = \sqrt{2g\Delta h}$$

$$v_{\text{bottom}} = \sqrt{2(9.81\tfrac{\text{m}}{\text{s}^2})(14.6\text{ m})}$$

$$v_{\text{bottom}} = \pm 16.925\ \frac{\text{m}}{\text{s}}$$

Apply the conservation of momentum to find the velocity of the boulder–shack system immediately after the collision.

$$m_{\text{b}}\vec{v}_{\text{b}} + m_{\text{s}}\vec{v}_{\text{s}} = m_{\text{b}}\vec{v}_{\text{b}}' + m_{\text{s}}\vec{v}_{\text{s}}'$$

$$m_{\text{b}}\vec{v}_{\text{b}} + 0 = (m_{\text{b}} + m_{\text{s}})\vec{v}_{\text{bs}}'$$

$$\vec{v}_{\text{bs}}' = \frac{m_{\text{b}}\vec{v}_{\text{b}}}{(m_{\text{b}} + m_{\text{s}})}$$

$$\vec{v}_{\text{bs}}' = \frac{(55.6\text{ kg})(16.925\text{ m/s[forward]})}{(55.6\text{ kg} + 204\text{ kg})}$$

$$\vec{v}_{\text{bs}}' = 3.6249\text{ m/s[forward]}$$

$$\vec{v}_{\text{bs}}' \cong 3.62\text{ m/s[forward]}$$

The velocity of the boulder–shack system immediately after the collision is about 3.62 m/s[forward].

Due to the law of conservation of energy, the work done on the boulder–shack system by the force of friction is equal to the kinetic energy of the system after the collision.

$$W_{\text{(to stop system)}} = E_{\text{k(after collision)}}$$

Substitute the expressions for work and kinetic energy into the equations. Both quantities (work and kinetic energy) are scalars, so only the magnitudes of the displacement and velocity are needed.

$$F\Delta d = \frac{1}{2}m_{\text{bs}}v'^2_{\text{bs}}$$

Friction is the force doing the work, and it is always parallel to the direction of motion. Substitute the formula for the force of friction.

$$F_{\text{f}}\Delta d = \frac{1}{2}m_{\text{bs}}v'^2_{\text{bs}}$$

$$\mu F_{\text{N}}\Delta d = \frac{1}{2}m_{\text{bs}}v'^2_{\text{bs}}$$

Since the system is moving horizontally, the normal force is the weight of the boulder–shack. Substitute the weight into the expression and solve for the displacement.

$$\mu m_{bs} g \Delta d = \frac{1}{2} m_{bs} v'^2_{bs}$$

$$\Delta d = \frac{12 v'^2_{bs}}{\mu g}$$

$$\Delta d = \frac{\frac{1}{2}(3.625 \text{ m/s})^2}{(0.392)(9.81 \text{ m/s})^2}$$

$$\Delta d = 1.7085 \text{ m}$$

$$\Delta d \cong 1.71 \text{ m}$$

The shack and boulder slide about 1.71 m across the ice.

Validate the Solution

The units for the distance are metres, as required. The shack is nudged only 1.7 m by the boulder, which seems reasonable.

Chapter 10 Review

Answers to Problems for Understanding

Student Textbook pages 180–181

23. (a) The kayak should be pointed 54° upstream. The unknown velocity vector is the hypotenuse of a triangle whose other sides are known.

$$\sin\theta = \frac{|\vec{v}_{ws}|}{|\vec{v}_{kw}|}$$

$$\theta = \sin^{-1}\frac{2.1 \text{ m/s}}{2.6 \text{ m/s}}$$

$$\theta = 53.87°$$

$$\theta \cong 54°$$

(b) The kayak's velocity relative to the shore will be 1.5 m/s.
Two of the three sides of a right triangle are known. Use the Pythagorean theorem to determine the third side.

$$|\vec{v}_{ks}|^2 + |\vec{v}_{ws}|^2 = |\vec{v}_{kw}|^2$$

$$|\vec{v}_{ks}|^2 = |\vec{v}_{kw}|^2 - |\vec{v}_{ws}|^2$$

$$|\vec{v}_{ks}|^2 = (2.6 \text{ m/s})^2 - (2.1 \text{ m/s})^2$$

$$|\vec{v}_{ks}|^2 = 2.35 \text{ (m/s)}^2$$

$$|\vec{v}_{ks}| = 1.533 \text{ m/s}$$

$$|\vec{v}_{ks}| \cong 1.5 \text{ m/s}$$

(c) It will take 29 s to paddle across.
Use the definition of velocity,

$$\vec{v}_{ks} = \frac{\Delta\vec{d}}{\Delta t}$$

$$\Delta t = \frac{\Delta d}{\vec{v}_{ks}}$$

$$\Delta t = \frac{45 \text{ m}}{1.533 \text{ m/s}} = 29.35 \text{ s}$$

$$\Delta t \cong 29 \text{ s}$$

24. (a) The fourth vector would be 1.0×10^2 N[E27°S]

The problem is done exactly like practice problem number 7.

(b) The fourth vector would be 34 N[E89°S].

The problem is done exactly like practice problem number 7.

(c) The fifth force would be 1.4×10^2 N[E66.8°N].

The problem is done exactly like practice problem number 7.

25. The coefficient of kinetic friction would be 0.16.

Find the net force of the three dogs in the forward x-direction.

F_{1x} = (83 N)cos15.5° = 79.98 N
F_{2x} = (75 N)cos9.0° = 74.08 N
F_{3x} = (77 N)cos12.0° = 75.32 N

The x-component of the forward force is

F_{dogsx} = 79.98 N + 93.83 N + 75.317 N = 249.13 N

Calculate the y-component:

F_{1y} = (83 N)sin15.5° = 22.18 N
F_{2y} = (75 N)sin9.0° = –11.732 N
F_{3y} = (77 N)sin12.0° = –16.009 N

The y-component of the forward force is

$$F_{dogsy} = 22.18 \text{ N} - 11.732 \text{ N} - 16.009 \text{ N} = 5.561 \text{ N}$$

$$F_{dogs} = \sqrt{F_{dogsx}^2 + F_{dogsy}^2}$$

$$F_{dogs} = \sqrt{(229.377 \text{ N})^2 + (5.561 \text{ N})^2}$$

$$F_{dogs} = 229.44 \text{ N}$$

Apply Newton's second law. Since the sled has constant velocity, it's acceleration is zero. The only force opposing the force of the dogs, is the force of friction.

$$F_{dogs} - F_f = ma_x = 0$$

$$F_{dogs} = F_f$$

$$F_{dogs} = \mu_k F_N$$

$$F_{dogs} = \mu_k mg$$

$$\mu_k = \frac{F_{dogs}}{mg}$$

$$\mu_k = \frac{229.44 \text{ N}}{(145 \text{ kg})(9.81 \text{ m/s}^2)}$$

$$\mu_k = 0.16$$

26. No, they will not move.

(Note: the free body diagrams for this question are the same as for Practice Problem #27.)

First, find the forces parallel to the plane. The force of gravity for mass 2 acts opposite to the combination of the force of friction and the parallel component of the force of gravity for the block.

To find the frictional force on m_1, apply Newton's second law in the direction perpendicular to the plane.

$F_\perp = ma_\perp = 0$
$F_N - F_{g1\perp} = 0$
$F_N = m_1 g\cos\theta$

$F_f = \mu_s F_N$
$F_f = \mu_s m_1 g\cos\theta$
$F_f = (0.42)(47\text{ kg})(9.81\text{ m/s}^2)\cos25°$
$F_f = 175.50\text{ N}$

$F_{g1} = m_1 g\sin\theta$
$F_{g1} = (47\text{ kg})(9.81\text{ m/s}^2)\sin25°$
$F_{g1} = 194.86\text{ N}$

$F_{g2} = m_2 g$
$F_{g2} = (35\text{ kg})(9.81\text{ m/s}^2)$
$F_{g2} = 343.35\text{ N}$

$F_{g1} + F_f = (194.86\text{ N}) + (175.50\text{ N}) = 370.35$

Since $F_{g1} + F_f > F_{g2}$, the masses will not move.

(b) N/A

(c) You need to add 2.8 kg to m_2 to cause the masses to begin to move.
Set $F_{g2} = 370.35$ N, i.e. the sum of F_{g1} and F_f, and determine the mass that this force implies.

$F_{g2} = m_2' g = 370.35\text{ N}$

$$m_2' = \frac{370.35\text{ N}}{9.81\text{ m/s}^2} = 37.75\text{ kg}$$

Now determine the difference between this mass and the given mass:

$\Delta m = m_2' - m_2 = 37.75\text{ kg} - 35\text{ kg} = 2.75\text{ kg}$

$\Delta m \cong 2.8\text{ kg}$

(d) The acceleration of the masses would be 1.1 m/s^2.
Apply Newton's law in the x-direction (parallel to the plane):

$F_x = ma_x$

$F_{g2x} - F_{g1x} - F_f = (m_1 + m_2)a_x$

$m_2 g - m_1 g\sin\theta - \mu_k F_N = (m_1 + m_2)a_x$

$$a_x = \frac{m_2 g - m_1 g\sin\theta - \mu_k(m_1 g\sin\theta)}{(m_1 + m_2)}$$

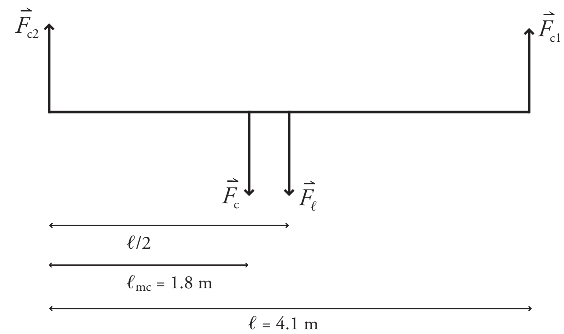
$$a_x = \frac{m_2 - m_1\sin\theta - \mu_k m_1\sin\theta}{(m_1 + m_2)} g$$

$$a_x = \frac{37.75\text{ kg} - (47\text{ kg})\sin25° - (0.19)(47\text{ kg})\cos25°}{(47\text{ kg} + 37.75\text{ kg})} \times 9.81\text{ m/s}^2$$

$a_x = 1.134\text{ m/s}^2$

$a_x \cong 1.1\text{ m/s}^2$

27. The left side of the crevasse is pushing up on the ladder with $F_{c1} = 3.9 \times 10^2$ N and the right side of the crevasse is pushing up on the ladder with $F_{c2} = 5.0 \times 10^2$ N.



Consider the far side of the crevasse, c1, as the axis.

$$\Sigma\tau = 0$$

$$-\frac{l}{2}F_l - l_{mc}F_{mc} + lF_{c2} = 0$$

$$F_{c2} = \frac{\frac{l}{2}F_l + l_{mc}F_{mc}}{l}$$

$$F_{c2} = \frac{\frac{4.1\text{ m}}{2}(3.6\text{ kg})(9.81\text{ m/s}^2) + (1.8\text{ m})(87\text{ kg})(9.81\text{ m/s}^2)}{4.1\text{ m}}$$

$$F_{c2} = 392.35\text{ N}$$

$$F_{c2} \cong 3.9 \times 10^2\text{ N}$$

$$\Sigma F = 0$$

$$F_{c1} + F_{c2} - F_{mc} - F_l = 0$$

$$F_{c1} = F_{mc} + F_l - F_{c2}$$

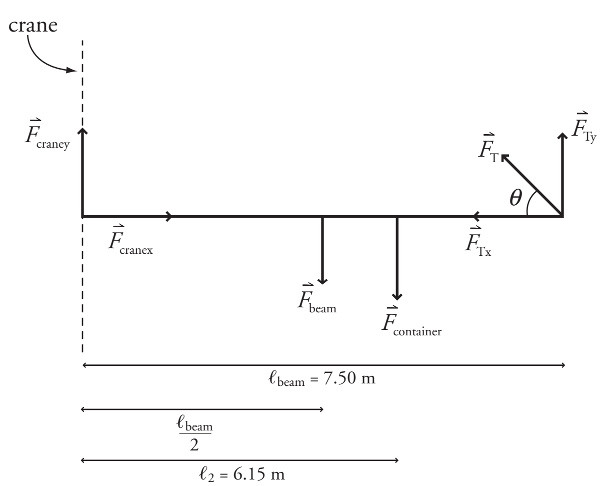
$$F_{c1} = m_{mc}g + m_l g - F_{c2}$$

$$F_{c1} = (87\text{ kg})(9.81\text{ m/s}^2) + (3.6\text{ kg})(9.81\text{ m/s}^2) - 392.35\text{ N}$$

$$F_{c1} = 496.44\text{ N}$$

$$F_{c1} \cong 5.0 \times 10^2\text{ N}$$

28.



(a) The tension in the arm is 8.58×10^3 N.

$\Sigma\tau = 0$ (choose the intersection of the beam with the crane pole as the axis

$$l_{beam}F_T \sin\theta - l_2F_{container} - \frac{l_{beam}}{2}F_{beam} = 0$$

$$l_{beam}F_T \sin\theta = l_2F_{container} + \frac{l_{beam}}{2}F_{beam}$$

$$F_T = \frac{l_2F_{container} + \frac{l_{beam}}{2}F_{beam}}{l_{beam}\sin\theta}$$

$$F_T = \frac{(6.15 \text{ m})(355 \text{ kg})(9.81 \text{ m/s}^2) + (\frac{7.50 \text{ m}}{2})(345 \text{ kg})(9.81 \text{ m/s}^2)}{(7.50 \text{ m})\sin 32^\circ}$$

$$F_T = 8582.28 \text{ N}$$

$$F_T \cong 8.58 \times 10^3 \text{ N}$$

(b) The crane exerts a force of 7.64×10^3 N[17.7° up from the beam].

$$\Sigma F_x = 0$$

$$F_{cranex} - F_{Tx} = 0$$

$$F_{cranex} = F_T \cos 32^\circ$$

$$F_{cranex} = (8582.28 \text{ N})\cos 32^\circ$$

$$F_{cranex} = 7278.19 \text{ N}$$

$$\Sigma F_y = 0$$

$$F_{craney} - F_{beam} - F_{container} + F_{Ty} = 0$$

$$F_{craney} = F_{beam} + F_{container} - F_{Ty}$$

$$F_{craney} = (345 \text{ kg})(9.81 \text{ m/s}^2) + (355 \text{ kg})(9.81 \text{ m/s}^2) - (8582.28 \text{ N})\sin 32^\circ$$

$$F_{craney} = 2319.08 \text{ N}$$

Apply the Pythagorean theorem.

$$\left|\vec{F}_{crane}\right|^2 = F_{cranex}^2 + F_{craney}^2$$

$$\left|\vec{F}_{crane}\right|^2 = (7278.19 \text{ N})^2 + (2319.08 \text{ N})^2$$

$$\left|\vec{F}_{crane}\right|^2 = 58350181.7 \text{ N}^2$$

$$\left|\vec{F}_{crane}\right| = 7638.73 \text{ N}$$

$$\left|\vec{F}_{crane}\right| \cong 7.64 \times 10^3 \text{ N}$$

Apply the tangent function:

$$\tan\theta = \frac{F_{craney}}{F_{cranex}}$$

$$\theta = \tan^{-1}\frac{2319.08 \text{ N}}{7278.19 \text{ N}}$$

$$\theta = \tan^{-1} 0.318634$$

$$\theta = 17.674^\circ$$

$$\theta \cong 17.7^\circ$$

29. Velocity and direction of the eight ball: Assume that "forward," the original direction of the cue ball, was in the positive y direction.

Momentum in the x direction

$$m_c v_{cx} + m_8 v_{8x} = m_c v'_{cx} + m_8 v'_{8x}$$
$$m_8 v'_{8x} = m_c v_{cx} - m_c v'_{cx} + m_8 v_{8x}$$
$$v'_{8x} = \frac{m_c(v_{cx} - v'_{cx})}{m_8} + \frac{m_8 v_{8x}}{m_8}$$
$$v'_{8x} = \frac{0.165\text{ kg}(0 - 3.7\ \frac{\text{m}}{\text{s}} \sin 40°)}{0.155\text{ kg}} + 0$$
$$v'_{8x} = -2.532\ \frac{\text{m}}{\text{s}}$$

Momentum in the y direction

$$m_c v_{cy} + m_8 v_{8y} = m_c v'_{cy} + m_8 v'_{8y}$$
$$m_8 v'_{8y} = m_c v_{cy} - m_c v'_{cy} + m_8 v_{8y}$$
$$v'_{8y} = \frac{m_c(v_{cy} - v'_{cy})}{m_8} + \frac{m_8 v_{8y}}{m_8}$$
$$v'_{8y} = \frac{0.165\text{ kg}(6.2\ \frac{\text{m}}{\text{s}} - 3.7\ \frac{\text{m}}{\text{s}} \cos 40°)}{0.155\text{ kg}} + 0$$
$$v'_{8y} = 3.583\ \frac{\text{m}}{\text{s}}$$

Pythagorean theorem and tangent function

$$\left|\vec{v}_8{}'\right|^2 = v'^2_{8x} + v'^2_{8y}$$
$$\left|\vec{v}_8{}'\right|^2 = \left(3.583\ \frac{\text{m}}{\text{s}}\right)^2 + \left(-2.532\ \frac{\text{m}}{\text{s}}\right)^2$$
$$\left|\vec{v}_8{}'\right|^2 = 12.838\ \frac{\text{m}^2}{\text{s}^2} + 6.411\ \frac{\text{m}^2}{\text{s}^2}$$
$$\left|\vec{v}_8{}'\right|^2 = 19.249\ \frac{\text{m}^2}{\text{s}^2}$$
$$\left|\vec{v}_8{}'\right| \cong 4.4\ \frac{\text{m}}{\text{s}}$$

$$\tan\theta = \frac{v'_{8y}}{v'_{8x}}$$

Negative x and positive y place the angle in the second quadrant.

$$\tan\theta = \frac{3.583\ \frac{\text{m}}{\text{s}}}{2.532\ \frac{\text{m}}{\text{s}}}$$
$$\theta = \tan^{-1}(1.4151)$$
$$\theta = 54.75°$$
$$\theta \cong 55° \text{ clockwise from negative } x\text{-axis}$$

30. (a) The final velocity of the blue ball is 0.29 m/s[W21°N].

Momentum in x direction

$$m_r v_{rx} + m_b v_{bx} = m_r v'_{rx} + m_b v'_{bx}$$
$$m_b v'_{bx} = m_r v_{rx} - m_r v'_{rx} + m_b v_{bx}$$
$$v'_{bx} = \frac{m_r(v_{rx} - v'_{rx})}{m_b} + \frac{m_b v_{bx}}{m_b}$$
$$v'_{bx} = \frac{0.750\text{ kg}\left(0.30\ \frac{\text{m}}{\text{s}}[\text{E}] - 0.15\ \frac{\text{m}}{\text{s}} \cos 30°[\text{E}]\right)}{0.550\text{ kg}} + 0.5\ \frac{\text{m}}{\text{s}}[\text{W}]$$
$$v'_{bx} = 0.232\ \frac{\text{m}}{\text{s}}[\text{E}] + 0.5\ \frac{\text{m}}{\text{s}}[\text{W}]$$
$$v'_{bx} = 0.268\ \frac{\text{m}}{\text{s}}[\text{W}]$$

Momentum in y direction

$$m_r v_{ry} + m_b v_{by} = m_r v'_{ry} + m_b v'_{by}$$
$$m_b v'_{by} = m_r v_{ry} - m_r v'_{ry} + m_b v_{by}$$
$$v'_{by} = \frac{m_r(v_{ry} - v'_{ry})}{m_b} + \frac{m_b v_{by}}{m_b}$$
$$v'_{by} = \frac{0.750\text{ kg}(0\ \frac{\text{m}}{\text{s}}[\text{E}]) - 0.15\ \frac{\text{m}}{\text{s}} \sin 30°[\text{S}]}{0.550\text{ kg}} + 0$$
$$v'_{by} = -0.1023\ \frac{\text{m}}{\text{s}}[\text{S}]$$
$$v'_{by} = 0.1023\ \frac{\text{m}}{\text{s}}[\text{N}]$$

Pythagorean theorem and tangent function

$$\left|\vec{v}_b'\right|^2 = v_{bx}'^2 + v_{by}'^2$$
$$\left|\vec{v}_b'\right|^2 = \left(-0.268\ \tfrac{m}{s}\right)^2 + \left(0.1023\ \tfrac{m}{s}\right)^2$$
$$\left|\vec{v}_b'\right|^2 = 0.082\,29\ \tfrac{m^2}{s^2}$$
$$\left|\vec{v}_b'\right| \cong 0.29\ \tfrac{m}{s}$$

$$\tan\theta = \frac{v'_{by}}{v'_{bx}}$$

A positive y- and negative x-component place the resultant vector in the second quadrant.

$$\tan\theta = \frac{0.1023\ \frac{m}{s}}{0.268\ \frac{m}{s}}$$
$$\theta = \tan^{-1}(0.381\,72)$$
$$\theta \cong 21^\circ$$

(b) Kinetic energy lost in the collision of the red ball and the blue ball: 69%

Calculate the kinetic energy before the collision.

$$E_{kr} = \frac{1}{2} m_r v_r^2$$
$$E_{kr} = \frac{1}{2}(0.750\ \text{kg})\left(0.30\ \tfrac{m}{s}\right)^2$$
$$E_{kr} = 0.033\,75\ \text{J}$$
$$E_{kb} = \frac{1}{2} m_b v_b^2$$
$$E_{kb} = \frac{1}{2}(0.550\ \text{kg})\left(0.50 \tfrac{m}{s}\right)^2$$
$$E_{kb} = 0.068\,75\ \text{J}$$
$$E_{kr} + E_{kb} = 0.1025\ \text{J}$$

Calculate the kinetic energy after the collision.

$$E_{kr}' = \frac{1}{2} m_r v_r^2$$
$$E_{kr}' = \frac{1}{2}(0.750\ \text{kg})\left(0.15\ \tfrac{m}{s}\right)^2$$
$$E_{kr}' = 0.008\,44\ \text{J}$$
$$E_{kb}' = \frac{1}{2} m_b v_b^2$$
$$E_{kb}' = \frac{1}{2}(0.550\ \text{kg})\left(0.29 \tfrac{m}{s}\right)^2$$
$$E_{kb}' = 0.023\,13\ \text{J}$$
$$E_{kr}' + E_{kb}' = 0.031\,57\ \text{J}$$

Calculate percent total kinetic energy lost.

$$\left(\frac{0.1025\ \text{J} - 0.031\,57\ \text{J}}{0.1025\ \text{J}}\right) \times 100\%$$
$$\cong 69\%$$