Chapter 12

Universal Gravitation

Practice Problem Solutions

Student Textbook page 580

1. **Conceptualize the Problem**
   - The law of universal gravitation applies to this problem.

   Identify the Goal
   The gravitational force, F_g, between Earth and the Sun

   Identify the Variables

Known	Implied	Unknown
$M_S = 1.99 \times 10^{30}$ kg	$G = 6.67 \times 10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}$	F_g
$M_E = 5.98 \times 10^{24}$ kg		
$r = 1.49 \times 10^{11}$ m		

Develop a Strategy	Calculations
Apply the law of universal gravitation.	$F_g = G\frac{m_1 m_2}{r^2}$
Substitute the numerical values and solve.	$F_g = \left(6.67 \times 10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)\frac{(1.99 \times 10^{30}\ \text{kg})(5.98 \times 10^{24}\ \text{kg})}{(1.49 \times 10^{11}\ \text{m})^2}$ $F_g = 3.575 \times 10^{22}$ N $F_g \cong 3.58 \times 10^{22}$ N

   The gravitational force between Earth and the Sun is about 3.58×10^{22} N.

   Validate the Solution
   Gravitational force causes Earth to go around the Sun, so it's expected that the force will be large, and it is.

2. **Conceptualize the Problem**
   - The law of universal gravitation applies to this problem.

   Identify the Goal
   The gravitational force, F_g, between Earth and the Moon

   Identify the Variables

Known	Implied	Unknown
$M_M = 7.36 \times 10^{22}$ kg	$G = 6.67 \times 10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}$	F_g
$M_E = 5.98 \times 10^{24}$ kg		
$r = 3.84 \times 10^{8}$ m		

Develop a Strategy	Calculations
Apply the law of universal gravitation.	$F_g = G\frac{m_1 m_2}{r^2}$

Substitute the numerical values and solve.

$$F_g = \left(6.67 \times 10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)\frac{(7.36 \times 10^{22}\ \text{kg})(5.98 \times 10^{24}\ \text{kg})}{(3.84 \times 10^{8}\ \text{m})^2}$$
$$F_g = 1.9909 \times 10^{20}\ \text{N}$$
$$F_g \cong 1.99 \times 10^{20}\ \text{N}$$

The gravitational force between Earth and Moon is about 1.99×10^{20} N.

Validate the Solution
Gravitational force causes the Moon to orbit Earth, so it's expected to be large, though smaller than in the previous problem, and it is.

3. Conceptualize the Problem
- The law of universal gravitation applies to this problem.

Identify the Goal
The distance, r, between two bowling balls that feel a given force between them

Identify the Variables

Known	Implied	Unknown
$m_1 = m_2 = 7.0$ kg	$G = 6.67 \times 10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}$	r
$F_g = 1.25 \times 10^{-4}$ N		

Develop a Strategy	Calculations
Apply the law of universal gravitation.	$F_g = G\frac{m_1 m_2}{r^2}$
Solve for the distance.	$r^2 = G\frac{m_1 m_2}{F_g}$
	$r = \sqrt{\frac{G m_1 m_2}{F_g}}$
Substitute the numerical values and solve.	$r = \sqrt{\frac{\left(6.67 \times 10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)(7.0\ \text{kg})^2}{1.25 \times 10^{-4}\ \text{N}}}$
	$r = 5.113 \times 10^{-3}$ m
	$r \cong 5.1 \times 10^{-3}$ m

The distance between the two bowling balls that feel the given force is about 5.1×10^{-3} m.

The distance in the law of universal gravitation is measured from the centre of an object. This calculated distance, about 0.5 cm, is smaller than the radius of a bowling ball, so it would not be possible to place them at this distance.

Validate the Solution
The units work out to be metres, as required.

4. Conceptualize the Problem
- The law of universal gravitation applies to this problem.

Identify the Goal
The gravitational force, F_g, between the electron and proton in a hydrogen atom

Identify the Variables

Known	Implied	Unknown
$m_p = 1.67 \times 10^{-27}$ kg	$G = 6.67 \times 10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}$	F_g
$m_e = 1.67 \times 10^{-27}$ kg		
$r = 5.30 \times 10^{-11}$ m		

Develop a Strategy **Calculations**

Apply the law of universal gravitation.

$$F_g = G\frac{m_1 m_2}{r^2}$$

Substitute the numerical values and solve.

$$F_g = \left(6.67 \times 10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)\frac{(1.67 \times 10^{-27}\ \text{kg})(9.11 \times 10^{-31}\ \text{kg})}{(5.30 \times 10^{-11}\ \text{m})^2}$$

$$F_g = 3.6125 \times 10^{-47}\ \text{N}$$

$$F_g \cong 3.61 \times 10^{-47}\ \text{N}$$

The gravitational force is about 3.61×10^{-47} N.

Validate the Solution

The masses are very small, so the gravitational force is also expected to be small, and it is.

5. Conceptualize the Problem

- The force that provides the weight of the person is the same as the force in the law of universal gravitation.
- The law of universal gravitation applies to this problem.

Identify the Goal

The mass of Venus, M

Identify the Variables

Known	Implied	Unknown
$F_g = 572$ N	$G = 6.67 \times 10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}$	M
$m = 68$ kg		
$r = 6.31 \times 10^6$ m		

Develop a Strategy **Calculations**

Apply the law of universal gravitation.

$$F_g = G\frac{Mm}{r^2}$$

Substitute the numerical values and solve.

$$M = \frac{F_g r^2}{Gm}$$

$$M = \frac{(572\ \text{N})(6.31 \times 10^6\ \text{m})^2}{\left(6.67 \times 10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)(68\ \text{kg})}$$

$$M = 5.0213 \times 10^{24}\ \text{kg}$$

$$M \cong 5.0 \times 10^{24}\ \text{kg}$$

The mass of Venus is about 5.0×10^{24} kg.

Validate the Solution

The calculated value of the mass of Venus is about 84% of Earth's mass, which seems reasonable knowing that the two planets have similar masses.

6. Conceptualize the Problem

- The law of universal gravitation applies to this problem.

Identify the Goal

The distance, r, between the two objects

Identify the Variables

Known	Implied	Unknown
$F_g = 1.28 \times 10^{-8}$ N	$G = 6.67 \times 10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}$	r
$m_1 = 8.0$ kg		
$m_2 = 1.5$ kg		

Develop a Strategy / **Calculations**

Apply the law of universal gravitation.
$$F_g = G\frac{m_1 m_2}{r^2}$$

Substitute the numerical values and solve.
$$r^2 = G\frac{m_1 m_2}{F_g}$$
$$r = \sqrt{\frac{Gm_1 m_2}{F_g}}$$
$$r = \sqrt{\frac{\left(6.67 \times 10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)(8.0\ \text{kg})(1.5\ \text{kg})}{1.28 \times 10^{-8}\ \text{N}}}$$
$$r = 0.2501\ \text{m}$$
$$r \cong 0.25\ \text{m}$$

The separation between the centres of the two objects is about 0.25 m.

Validate the Solution

The distance between the two objects seems reasonable for a laboratory experiment.

7. Conceptualize the Problem

- The law of universal gravitation applies to this problem.
- This problem can be solved using ratios.

Identify the Goal

The gravitational force on Uranus, F_U, compared to that on Earth, F_E

Identify the Variables

Known	Implied	Unknown
$r_U = 4.3r_E$	$G = 6.673 \times 10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}$	F_U
$M_U = 14.7M_E$		

Develop a Strategy / **Calculations 1** / **Calculations 2 (alternate method)**

Apply the law of universal gravitation.

Calculations 1:
$$F_U = G\frac{M_U m}{r_U^2}$$
$$F_E = G\frac{M_E m}{r_E^2}$$

Divide one by the other.
$$\frac{F_U}{F_E} = \frac{GM_U m}{r_U^2} \times \frac{r_E^2}{GM_E m}$$
$$\frac{F_U}{F_E} = \frac{M_U}{M_E} \times \frac{r_E^2}{r_U^2}$$

Substitute the numerical values and solve.
$$\frac{F_U}{F_E} = \frac{(14.7M_E)}{M_E} \times \frac{r_E^2}{(4.3r_E)^2}$$
$$\frac{F_U}{F_E} = \frac{14.7}{4.3^2}$$
$$\frac{F_U}{F_E} = 0.795$$
$$F_U \cong 0.80F_E$$

Calculations 2 (alternate method):
$$F_U = G\frac{M_U m}{r_U^2}$$
$$F_U = G\frac{(14.7M_E)m}{(4.3r_E)^2}$$
$$F_U = \left(\frac{14.7}{4.3^2}\right)\left(G\frac{M_E m}{r_E^2}\right)$$
$$F_U = 0.795F_E$$
$$F_U \cong 0.80F_E$$

The gravitational force on the surface of Uranus is about 0.80 times the gravitational force on the surface of Earth.

Validate the Solution

Both the mass and radius of Uranus are larger than that of Earth. However, because the law of gravitation depends on the square of the radius, the gravitational force on the surface of Uranus is smaller than that of Earth.

8. Conceptualize the Problem

- The object is to be placed where it would feel equal *gravitational forces* from Earth and the Moon so that the *net force* on the object is zero.
- The law of universal gravitation applies to this problem.
- Make a sketch of the problem and indicate the distances. Let the Earth-Moon distance be *d*, the distance from the Earth to the point be *x* and the distance from the Moon to the point be *d-x*.

Identify the Goal

The distance, *x*, from the centre of Earth, where a point between Earth and the Moon would feel equal and opposite forces

Identify the Variables

Known	Implied	Unknown
$M_M = 0.0123 \times M_E$	$G = 6.673 \times 10^{-11}\ \frac{N \cdot m^2}{kg^2}$	x
		F_M
		F_E

Develop a Strategy

Apply Newton's second law to a test mass placed at the point.

Calculations

$$F_{net} = F_E - F_M = ma = 0$$
$$F_E = F_M$$
$$G\frac{M_E m}{x^2} = G\frac{M_M m}{(d-x)^2}$$
$$(d-x)^2 = \frac{M_M}{M_E}x^2$$
$$(d-x)^2 - \frac{M_M}{M_E}x^2 = 0$$

Let $d = 1.0$. Then the distance, *x*, will be quoted as a fraction of the Earth-Moon distance.

$$d^2 - 2dx + x^2 - \frac{M_M}{M_E}x^2 = 0$$
$$x^2\left(1 - \frac{M_M}{M_E}\right) - 2dx + d^2 = 0$$

A quadratic equation is obtained which can be solved using the quadratic formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$0.9877x^2 - 2x + 1 = 0$$
$$x = \frac{-(-2) \pm \sqrt{2^2 - 4(0.9877)(1)}}{2(0.9877)}$$

Choose the solution that is less than 1.0 (because *x* should be less than *d*).

$$x = 0.900 \text{ or } 1.124$$
$$x \cong 0.900$$

A point 0.9 times the Earth-Moon distance will feel equal and opposite gravitational forces from Earth and the Moon.

Validate the Solution

The Earth is much more massive than the Moon, so it's expected that the point where the gravitational forces cancel will be closer to the Moon than to Earth.

Practice Problem Solutions

Student Textbook page 586

9. Conceptualize the Problem

- Kepler's third law, combined with Newton's law of universal gravitation, yields an equation that relates the period and orbital radius of a satellite to the mass of the body around which the satellite is orbiting.

Identify the Goal

The mass of Jupiter, M_J

Identify the Variables

Known	Implied	Unknown
$T = 1.769$ days	$G = 6.673 \times 10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}$	M_{J}
$r = 4.216 \times 10^8$ m		

Develop a Strategy	Calculations
Write Kepler's third law, using the constant derived from Newton's law of universal gravitation. Solve for the mass.	$\frac{T^2}{r^3} = \frac{4\pi^2}{GM_{\text{J}}}$ $M_{\text{J}} = \frac{4\pi^2 r^3}{GT^2}$
Substitute the numerical values and solve.	$M_{\text{J}} = \frac{4\pi^2(4.216 \times 10^8\ \text{m})^3}{\left(6.673 \times 10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)\left(1.769\ \text{days} \times \frac{24\ \text{h}}{1\ \text{day}} \times \frac{3600\ \text{s}}{1\ \text{h}}\right)^2}$
Convert days to seconds.	$M_{\text{J}} = 1.898 \times 10^{27}$ kg

The mass of Jupiter is about 1.898×10^{27} kg.

Validate the Solution

From reference tables, the mass of Jupiter is about 317.8 times the mass of Earth. This is in excellent agreement $\left(\frac{1.898 \times 10^{27}\ \text{kg}}{317.8} = 5.972 \times 10^{24}\ \text{kg} \cong \text{mass of Earth}\right)$ with the above calculation.

10. Conceptualize the Problem

- Kepler's third law, combined with Newton's law of universal gravitation, yields an equation that relates the period and orbital radius of a satellite to the mass of the body around which the satellite is orbiting.

Identify the Goal

The mass of Pluto, M_{P}

Identify the Variables

Known	Implied	Unknown
$T = 6.387$ days	$G = 6.673 \times 10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}$	M_{P}
$r = 1.9640 \times 10^7$ m		

Develop a Strategy	Calculations
Write Kepler's third law, using the constant derived from Newton's law of universal gravitation. Solve for the mass.	$\frac{T^2}{r^3} = \frac{4\pi^2}{GM_{\text{P}}}$ $M_{\text{P}} = \frac{4\pi^2 r^3}{GT^2}$
Substitute the numerical values and solve.	$M_{\text{P}} = \frac{4\pi^2(1.9640 \times 10^7\ \text{m})^3}{\left(6.673 \times 10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)\left(6.387\ \text{days} \times \frac{24\ \text{h}}{1\ \text{day}} \times \frac{3600\ \text{s}}{1\ \text{h}}\right)^2}$
Convert days to seconds.	$M_{\text{P}} = 1.472 \times 10^{22}$ kg

The mass of Pluto is about 1.472×10^{22} kg.

Validate the Solution

From reference tables, the mass of Pluto is about 0.0026 times the mass of Earth. This gives a value of $\frac{1.472 \times 10^{22}\ \text{kg}}{0.0026} = 5.66 \times 10^{24}$ kg for the mass of Earth, within about 5.0% of the accepted value.

11. Conceptualize the Problem

- Kepler's third law, combined with Newton's law of universal gravitation, yields an equation that relates the period and orbital radius of a satellite to the mass of the body around which the satellite is orbiting.

Identify the Goal

The altitude, h, of the satellite above Earth's surface

Identify the Variables

Known	Implied	Unknown
$T = 90.0$ m	$G = 6.67 \times 10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}$	r
$M_E = 5.98 \times 10^{24}$ kg		h
$r_E = 6.38 \times 10^6$ m		

Develop a Strategy / **Calculations**

Write Kepler's third law, using the constant derived from Newton's law of universal gravitation. Solve for the orbital radius.

$$\frac{T^2}{r^3} = \frac{4\pi^2}{GM_E}$$

$$r^3 = \frac{GM_P T^2}{4\pi^2}$$

Substitute the numerical values and solve.
Convert minutes to seconds.

$$r = \left(\frac{\left(6.67 \times 10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)(5.98 \times 10^{24}\ \text{kg})\left(90.0\ \text{min} \times \frac{60\ \text{s}}{1\ \text{min}}\right)^2}{4\pi^2}\right)^{\frac{1}{3}}$$

$r = 6.654 \times 10^6$ m

The orbital radius is about 6.654×10^6 m.

Find the altitude above Earth's surface by subtracting Earth's radius from the orbital radius.

$$h = r - r_E$$

$$h = 6.654 \times 10^6\ \text{m} - 6.38 \times 10^6\ \text{m}$$

$$h = 2.74 \times 10^5\ \text{m}$$

The satellite orbits at 2.74×10^5 m above Earth's surface, or, about 274 km.

Validate the Solution

Knowing that Earth's mesosphere extends up to 80 km, and the upper atmosphere has a much lower density, the result is reasonable.

Check the units: $\left(\frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}(\text{kg})(\text{s})^2\right)^{\frac{1}{3}} = \left(\frac{\frac{\text{kg}\cdot\text{m}}{\text{s}^2}\text{m}^2}{\text{kg}^2}(\text{kg})(\text{s})^2\right)^{\frac{1}{3}} = (\text{m}^3)^{\frac{1}{3}} = \text{m}$

12. Conceptualize the Problem

- The *gravitational force* between Earth and Moon provides the *centripetal force* to keep the Moon in its orbit.
- Assume the Moon is orbiting Earth in a circle.

Identify the Goal

The speed of the Moon, v, as it orbits Earth

Identify the Variables

Known	Implied	Unknown
$T = 90.0\ \text{m}$	$G = 6.67 \times 10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}$	v
$M_E = 5.98 \times 10^{24}\ \text{kg}$		
$r_M = 3.84 \times 10^5\ \text{km}$ $= 3.84 \times 10^8\ \text{m}$		

Develop a Strategy

Calculations

Apply the law of universal gravitation with Newton's second law, noting that the acceleration is centripetal acceleration.

$$F = G\frac{M_E m_M}{r^2} = m_M a = m_M \frac{v^2}{r}$$

$$G\frac{M_E}{r} = v^2$$

$$v = \sqrt{\frac{GM_E}{r}}$$

Substitute the numerical values and solve.

$$v = \sqrt{\frac{\left(6.67 \times 10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)(5.98 \times 10^{24}\ \text{kg})}{(3.84 \times 10^8\ \text{m})}}$$

Convert minutes to seconds.

$$v = 1.0192 \times 10^3\ \frac{\text{m}}{\text{s}}$$

$$v \cong 1.02 \times 10^3\ \frac{\text{m}}{\text{s}}$$

The orbital speed of the Moon is about $1.02 \times 10^3\ \frac{\text{m}}{\text{s}}\ \left(\text{or } 1.02\ \frac{\text{km}}{\text{s}}\right)$.

Validate the Solution

Comparing the result with Earth's orbital speed,

$v_E = \sqrt{\frac{GM_S}{r_{E(\text{orbit})}}} = \sqrt{\frac{\left(6.67 \times 10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)(1.99 \times 10^{30}\ \text{kg})}{(1.49 \times 10^{11}\ \text{m})}} = 29.8\ \frac{\text{km}}{\text{s}}$. Earth feels a significantly greater gravitational force from the Sun than the Moon does from Earth (see Practice Problems 1 and 2), so it is expected that Earth's orbital speed will be greater than the Moon's.

Check the units: $\left(\frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}(\text{kg})\left(\frac{1}{\text{m}}\right)\right)^{\frac{1}{2}} = \left(\frac{\frac{\text{kg}\cdot\text{m}}{\text{s}^2}\text{m}^2}{\text{kg}^2}(\text{kg})\left(\frac{1}{\text{m}}\right)\right)^{\frac{1}{2}} = \left(\frac{\text{m}^2}{\text{s}^2}\right)^{\frac{1}{2}} = \frac{\text{m}}{\text{s}}$

13. Conceptualize the Problem

- Kepler's third law, combined with Newton's law of universal gravitation, yields an equation that relates the period and orbital radius of a satellite to the mass of the body around which the satellite is orbiting.
- The *gravitational force* between the Moon and the *Apollo* lunar module provides the *centripetal force* to keep the module in its orbit.
- Assume the *Apollo* lunar module is orbiting the Moon in a circle.

Identify the Goal

(a) The orbital period, T, of the lunar module

(b) The orbital speed, v, of the lunar module

Identify the Variables

Known	Implied	Unknown
$M_M = 7.36 \times 10^{22}\ \text{kg}$	$G = 6.67 \times 10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}$	T
$h = 60.0\ \text{km}$		v
$r_M = 7738\ \text{km}$		

Develop a Strategy | **Calculations**

Note that the orbital radius is the altitude plus the Moon's radius. Write Kepler's third law, using the constant derived from Newton's law of universal gravitation.

$$\frac{T^2}{r^3} = \frac{4\pi^2}{GM_M}$$

$$T^2 = \frac{4\pi^2 r^3}{GM_M}$$

Solve for the orbital period.

$$T = \sqrt{\frac{4\pi^2 r^3}{GM_M}}$$

Substitute the numerical values and solve.

$$T = \sqrt{\frac{4\pi^2\left(60.0\text{ km} \times \frac{1000\text{ m}}{1\text{ km}} + 7738\text{ km} \times \frac{1000\text{ m}}{1\text{ km}}\right)^3}{\left(6.67 \times 10^{-11}\ \frac{\text{N·m}^2}{\text{kg}^2}\right)(7.36 \times 10^{22}\text{ kg})}}$$

$$T = 6.1752 \times 10^4\text{ s}$$

Multiply by the number of seconds in one hour to obtain the result in hours.

$$T = 6.1752 \times 10^4\text{ s} \times \frac{1\text{ h}}{3600\text{ s}}$$

$$T = 17.2\text{ h}$$

(a) The orbital period of the lunar module is about 6.18×10^4 s or about 17.2 h.

Apply the law of universal gravitation with Newton's second law, noting that the acceleration is centripetal acceleration.

$$F = G\frac{M_M m_{module}}{r^2} = m_{module}a = m_{module}\frac{v^2}{r}$$

$$G\frac{M_M}{r} = v^2$$

$$v = \sqrt{\frac{GM_M}{r}}$$

Substitute the numerical values and solve.

$$v = \sqrt{\frac{\left(6.67 \times 10^{-11}\ \frac{\text{N·m}^2}{\text{kg}^2}\right)(7.36 \times 10^{22}\text{ kg})}{(60.0 \times 10^3\text{ m} + 7738 \times 10^3\text{ m})}}$$

$$v = 7.93 \times 10^2\ \frac{\text{m}}{\text{s}}$$

(b) The module has an orbital speed of about $7.93 \times 10^2\ \frac{\text{m}}{\text{s}}$.

Validate the Solution

Check the units for the period: $\left(\frac{(\text{m})^3}{\left(\frac{\text{N·m}^2}{\text{kg}^2}\right)(\text{kg})}\right)^{\frac{1}{2}} = \left(\frac{\text{m}^3}{\frac{\text{kg·m}}{\text{s}^2}\frac{\text{m}^2}{\text{kg}^2}\text{kg}}\right)^{\frac{1}{2}} = (\text{s}^2)^{\frac{1}{2}} = \text{s}$

(The units for speed were checked in the previous question.)

14. Conceptualize the Problem

- The *gravitational force* between the centre of the galaxy and the star provides the *centripetal force* to keep the star in its orbit.
- Assume the star is orbiting the galaxy in a circle.

Identify the Goal

The mass of the Andromeda galaxy, M

Identify the Variables

Known	**Implied**	**Unknown**
$v = 2.0 \times 10^2\ \frac{\text{km}}{\text{s}}$	$G = 6.67 \times 10^{-11}\ \frac{\text{N·m}^2}{\text{kg}^2}$	M
$= 2.0 \times 10^5\ \frac{\text{m}}{\text{s}}$		
$r = 5 \times 10^9$ AU		

Develop a Strategy

Apply the law of universal gravitation with Newton's second law, noting that the acceleration is centripetal acceleration. Solve for the mass of the galaxy.

Substitute the numerical values and solve.

Calculations

$$F = G\frac{Mm_{star}}{r^2} = m_{star}a = m_{star}\frac{v^2}{r}$$

$$G\frac{M}{r} = v^2$$

$$M = \frac{rv^2}{G}$$

$$M = \frac{\left(5 \times 10^9 \text{ AU} \times \frac{1.49 \times 10^{11} \text{ m}}{1 \text{ AU}}\right)(2.0 \times 10^5 \frac{\text{m}}{\text{s}})^2}{\left(6.67 \times 10^{-11} \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)}$$

$$M = 4.4678 \times 10^{41} \text{ kg}$$

$$M \cong 4 \times 10^{41} \text{ kg}$$

The mass of the galaxy is about 4×10^{41} kg.

Validate the Solution

Check the units for the mass: $\frac{(\text{m})\left(\frac{\text{m}}{\text{s}}\right)^2}{\left(\frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)} = \frac{(\text{m})\left(\frac{\text{m}}{\text{s}}\right)^2}{\left(\frac{\text{kg}\cdot\text{m}}{\text{s}^2}\frac{\text{m}^2}{\text{kg}^2}\right)} = \text{kg}$

The mass of the galaxy can be compared to the mass of the Sun: $\frac{(4 \times 10^{41} \text{ kg})}{(2.0 \times 10^{30} \text{ kg})} = 2 \times 10^{11}$. Large galaxies are known to have masses of more than 100 billion times the sun's mass, so this result is reasonable.

Practice Problem Solutions

Student Textbook page 591

15. Conceptualize the Problem

- The period is related to the velocity of the satellite.
- The period of an orbit is the inverse of the frequency, f.
- The velocity and altitude of the satellite are determined by the amount of centripetal force that is causing the satellite to remain on a circular path.
- Earth's gravity provides the centripetal force for satellite motion.

Identify the Goal

The orbital speed, v, and altitude, h, of the satellite

Identify the Variables

Known	Implied	Unknown
$M_E = 5.98 \times 10^{24}$ kg	$G = 6.67 \times 10^{-11} \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}$	v
$f = 14.1$ orbits/day		h
		r

Develop a Strategy	Calculations
Determine the orbital period (in seconds) from the frequency (the number of orbits per day).	$T = \frac{1}{f}$ $T = \frac{1\text{ day} \times \frac{24\text{ h}}{1\text{ day}} \times \frac{3600\text{ s}}{1\text{ h}}}{14.1\text{ orbits}}$ $T = 6.1276 \times 10^3\text{ s}$
Write Kepler's third law, using the constant derived from Newton's law of universal gravitation. Solve for the orbital radius.	$\frac{T^2}{r^3} = \frac{4\pi^2}{GM_E}$ $r^3 = \frac{GM_E T^2}{4\pi^2}$

Substitute the numerical values and solve.

$$r = \left(\frac{\left(6.67 \times 10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)(5.98 \times 10^{24}\text{ kg})(6.1276 \times 10^3\text{ s})^2}{4\pi^2} \right)^{\frac{1}{3}}$$

$$r = 7.239 \times 10^6\text{ m}$$

$$r \cong 7.24 \times 10^6\text{ m}$$

Determine the altitude of the orbit by subtracting the radius of Earth from the orbital radius.	$h = r - r_E$ $h = 7.239 \times 10^6\text{ m} - 6.38 \times 10^6\text{ m}$ $h = 8.588 \times 10^5\text{ m}$ $h \cong 8.59 \times 10^5\text{ m}$
Apply the law of universal gravitation with Newton's second law, noting that the acceleration is centripetal acceleration.	$F = G\frac{M_E m_{satellite}}{r^2} = m_{satellite}a = m_{satellite}\frac{v^2}{r}$ $G\frac{M_E}{r} = v^2$ $v = \sqrt{\frac{GM_E}{r}}$
Substitute known values and solve.	$v = \sqrt{\frac{\left(6.67 \times 10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)(5.98 \times 10^{24}\text{ kg})}{(7.239 \times 10^6\text{ m})}}$ $v = 7.4229 \times 10^3\ \frac{\text{m}}{\text{s}}$ $v \cong 7.42 \times 10^3\ \frac{\text{m}}{\text{s}}$

The orbital speed of the satellite is about $7.42 \times 10^3\ \frac{\text{m}}{\text{s}}$.

Validate the Solution

Based on the result of problem 11, it is expected that a satellite that orbits Earth several times per day will have an altitude of several hundred km above Earth's surface, so the result here (859 km) seems reasonable.

16. Conceptualize the Problem

- The period is related to the velocity of the satellite.
- The velocity and altitude of the satellite are determined by the amount of centripetal force that is causing the satellite to remain on a circular path.
- Earth's gravity provides the centripetal force for satellite motion.

Identify the Goal

The orbital speed, v, and period, T, of the space station

Identify the Variables

Known	Implied	Unknown
$h = 226$ km	$G = 6.67 \times 10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}$	v
$M = 5.98 \times 10^{24}$ kg		T
$r_\text{E} = 6.38 \times 10^6$ m		

Develop a Strategy / Calculations

Apply the law of universal gravitation with Newton's second law, noting that the acceleration is centripetal acceleration.

$$F = G\frac{M_\text{E} m_\text{Space station}}{r^2} = m_\text{Space station}a = m_\text{Space station}\frac{v^2}{r}$$

$$G\frac{M_\text{E}}{r} = v^2$$

$$v = \sqrt{\frac{GM_\text{E}}{r}}$$

The orbital radius is the radius of Earth plus the altitude $r = r_\text{E} + h$.

$$v = \sqrt{\frac{\left(6.67 \times 10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)(5.98 \times 10^{24}\ \text{kg})}{(226 \times 10^3\ \text{m} + 6.38 \times 10^6\ \text{m})}}$$

$$v = 7.77 \times 10^3\ \frac{\text{m}}{\text{s}}$$

The Space Station has an orbital speed of about $7.77 \times 10^3\ \frac{\text{m}}{\text{s}}$.

Write Kepler's third law, using the constant derived from Newton's law of universal gravitation. Solve for the orbital period.

$$\frac{T^2}{r^3} = \frac{4\pi^2}{GM_\text{M}}$$

$$T^2 = \frac{4\pi^2 r^3}{GM_\text{M}}$$

$$T = \sqrt{\frac{4\pi^2 r^3}{GM_\text{M}}}$$

$$T = \sqrt{\frac{4\pi^2\left(226\ \text{km} \times \frac{1000\ \text{m}}{1\ \text{km}} + 6.38 \times 10^6\ \text{m}\right)^3}{\left(6.67 \times 10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)(5.98 \times 10^{24}\ \text{kg})}}$$

$$T = 5.342 \times 10^3\ \text{s}$$

Multiple by the number of seconds in an hour to obtain the period in hours.

$$T = 5.342 \times 10^3\ \text{s} \times \frac{1\ \text{h}}{3600\ \text{s}}$$

$$T = 1.48\ \text{h}$$

The orbital period of the satellite is about 5.34×10^3 s or 1.48 h.

Validate the Solution

The altitude and period of the International Space Station agree with the results of problem 11, so the results here are reasonable.

17. **Conceptualize the Problem**

- Kepler's third law relates the orbital radius and orbital period of a satellite or planet, to the mass of the object it orbits.
- The period is related to the velocity of the planet.
- The velocity of the planet is determined by the amount of centripetal force that is causing the planet to remain on a circular path.
- The Sun's gravity provides the centripetal force for planet motion.

Identify the Goal

(a) The orbital period, T and speed, v, of the planet Neptune

(b) The number of orbits, N, that Neptune has completed since its discovery

Identify the Variables

Known	Implied	Unknown
$M = 1.99 \times 10^{30}$ kg	$G = 6.67 \times 10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}$	v
$r_{\text{N}} = 4.50 \times 10^{12}$ m		T
		N

Develop a Strategy

Calculations

Write Kepler's third law, using the constant derived from Newton's law of universal gravitation. Solve for the orbital period.

$$\frac{T^2}{r^3} = \frac{4\pi^2}{GM_{\text{S}}}$$

$$T^2 = \frac{4\pi^2 r^3}{GM_{\text{S}}}$$

$$T = \sqrt{\frac{4\pi^2 r^3}{GM_{\text{S}}}}$$

$$T = \sqrt{\frac{4\pi^2(4.50 \times 10^{12}\ \text{m})^3}{\left(6.67 \times 10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)(1.99 \times 10^{30}\ \text{kg})}}$$

Multiply by the number of seconds in a year to obtain the number of years.

$$T = 5.21 \times 10^9\ \text{s} \times \frac{1\ \text{year}}{3.156 \times 10^7\ \text{s}}$$

$$T = 165\ \text{years}$$

(a) The orbital period of Neptune is about 5.21×10^9 s or 165 years.

Apply the law of universal gravitation with Newton's second law, noting that the acceleration is centripetal acceleration.

$$F = G\frac{M_{\text{S}}m_{\text{Neptune}}}{r^2} = m_{\text{Neptune}}a = m_{\text{Neptune}}\frac{v^2}{r}$$

$$G\frac{M_{\text{S}}}{r} = v^2$$

$$v = \sqrt{\frac{GM_{\text{S}}}{r}}$$

$$v = \sqrt{\frac{\left(6.67 \times 10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)(1.99 \times 10^{30}\ \text{kg})}{(4.50 \times 10^{12}\ \text{m})}}$$

$$v = 5.43 \times 10^3\ \frac{\text{m}}{\text{s}}$$

(a) The orbital speed of Neptune is about $5.43 \times 10^3\ \frac{\text{m}}{\text{s}}$.

(b) The period of Neptune is 165 years. It was discovered in 1846. Therefore, it will complete its first orbit since its discovery in the year: $1846 + 165 = 2011$.

Validate the Solution

According to reference tables, the period of Neptune is 165 years, in agreement with the above calculation.

Chapter 12 Review

Answers to Problems for Understanding

Student Textbook page 597

22. Reducing the mass by half will reduce the gravitational force by half also. Doubling the distance will reduce the force to one quarter. Together, the new force will be reduced to one eighth, or 10.0 N.

$$F_{gA} = G\frac{m_{1A}m_{2A}}{r_A^2}$$
$$F_{gA} = 80 \text{ N}$$
$$G\frac{m_{1A}m_{2A}}{r_A^2} = 80 \text{ N}$$
$$m_{1B} = \frac{1}{2}m_{1A} \quad m_{2B} = m_{2A} \quad r_B = 2r_A$$

$$F_{gB} = G\frac{m_{1B}m_{2B}}{r_B^2}$$
$$F_{gB} = G\frac{\frac{1}{2}m_{1A}m_{2A}}{(2r_A)^2}$$
$$F_{gB} = \frac{1}{8}F_{gA}$$
$$F_{gB} = \frac{1}{8}(80 \text{ N})$$
$$F_{gB} = 10 \text{ N}$$

23. The correct answer is (c) F. From Newton's third law: The gravitational force is equal on both stars.

24. The correct answer is (b) $\frac{a}{3}$. From Newton's second law: $a = \frac{F}{m}$.

$$F = ma \qquad F_1 = F_2$$
$$a = \frac{F}{m} \qquad a_2 = \frac{F_1}{3m^*}$$
$$a_1 = \frac{F_1}{m^*} \qquad a_2 = \left(\frac{1}{3}\right)\frac{F_1}{m^*}$$
$$a_2 = \frac{F_2}{3m^*} \qquad a_2 = \left(\frac{1}{3}\right)a_1$$

25. (a) From these equations, you obtain the expression $v = \sqrt{\frac{Gm}{r}}$, where m is the Sun's mass (1.99×10^{30} kg) and r is Earth's orbital radius (1.49×10^{11} m). Using this equation gives a velocity of 2.98×10^4 m/s.

$$F_g = F_c$$
$$G\frac{\cancel{m_E}m_g}{\cancel{r^2}} = \frac{\cancel{m_E}v^2}{\cancel{r}}$$

$$v = \sqrt{\frac{Gm}{r}}$$
$$v = \sqrt{\frac{\left(6.67 \times 10^{-11} \frac{\text{N}\cdot\text{m}^{\cancel{2}}}{\text{kg}^{\cancel{2}}}\right)\left(1.99 \times 10^{30} \cancel{\text{kg}}\right)}{1.49 \times 10^{11} \cancel{\text{m}}}}$$
$$v \cong 2.98 \times 10^4 \ \frac{\text{m}}{\text{s}}$$

(b) The equation for centripetal acceleration gives 5.98×10^{-3} m/s^2.

$$a_c = \frac{v^2}{r}$$
$$a_c = \frac{\left(2.9846 \times 10^4 \ \frac{\text{m}}{\text{s}}\right)^2}{1.49 \times 10^{11} \text{ m}}$$
$$a_c \cong 5.98 \times 10^{-3} \ \frac{\text{m}}{\text{s}^2}$$

26. From the law of universal gravitation, the force of gravity on the Sun from Earth is 3.56×10^{22} N. Using this force in Newton's second law, the Sun's acceleration is 1.80×10^{-8} m/s^2.

$$F_g = G\frac{m_1 m_2}{r^2}$$
$$F_g = \left(6.67 \times 10^{-11}\ \frac{\text{N}\cdot\cancel{\text{m}^2}}{\cancel{\text{kg}^2}}\right)\frac{(5.98 \times 10^{24}\ \cancel{\text{kg}})(1.99 \times 10^{30}\ \cancel{\text{kg}})}{(1.49 \times 10^{11}\ \cancel{\text{m}})^2}$$
$$F_g \cong 3.58 \times 10^{22}\ \text{N}$$

$$F = ma$$
$$a = \frac{F}{m}$$
$$a = \frac{3.5753 \times 10^{22}\ \text{N}}{1.99 \times 10^{30}\ \text{kg}}$$
$$a \cong 1.80 \times 10^{-8}\ \frac{\text{m}}{\text{s}^2}$$

27. By setting the force of gravity equal to *ma*, you obtain an expression for acceleration that yields $a = 8.95$ m/s^2. This value is only slightly less than the value of 9.81 m/s^2 at Earth's surface.

$$F_g = G\frac{m_E m_{SS}}{r^2}$$
$$m_{SS}a = G\frac{m_E m_{SS}}{r^2}$$
$$a = \left(6.67 \times 10^{-11}\frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)\frac{(5.98 \times 10^{24}\ \text{kg})}{(6.38 \times 10^6\ \text{m} + 2.95 \times 10^5\ \text{m})^2}$$
$$a \cong 8.95\ \frac{\text{m}}{\text{s}^2}$$

28. Setting the force of gravity equal to the centripetal force and solving for m_{bh} gives $m_{bh} = 4.1 \times 10^{36}$ kg — approximately two million times more massive than the Sun.

$$G\frac{m_{bh}\cancel{m_{gas}}}{r^{\cancel{2}}} = \frac{\cancel{m_{gas}}v^2}{\cancel{r}}$$
$$m_{bh} = \frac{v^2 r}{G}$$
$$m_{bh} = \frac{\left(3.4 \times 10^4\ \frac{\text{m}}{\text{s}}\right)^2(2.365 \times 10^{17}\ \text{m})}{6.67 \times 10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}}$$
$$m_{bh} \cong 4.1 \times 10^{36}\ \text{kg}$$

29. Using the law of universal gravitation, the force of gravity is 2.7×10^{-10} N. This value is in the order of one million times smaller than the weight of a flea.

$$F_g = G\frac{m_1 m_2}{r^2}$$
$$F_g = \left(6.67 \times 10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)\frac{(1.0\ \text{kg})(1.0\ \text{kg})}{(0.500\ \text{m})^2}$$
$$F_g \cong 2.7 \times 10^{-10}\ \text{kg}$$

30. (a) Setting the force of gravity equal to the centripetal force, solving for r, and subtracting Earth's radius of 6.38×10^6 m gives an altitude of 5.3×10^5 m.

$$G\frac{m_E m_H}{r^2} = \frac{m_H v^2}{r}$$

$$r = G\frac{m_E}{v^2}$$

$$r = \left(6.67 \times 10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)\frac{(5.98 \times 10^{24}\ \text{kg})}{\left(7.6 \times 10^3\ \frac{\text{m}}{\text{s}}\right)^2}$$

$$r = 6.9056 \times 10^6\ \text{m}$$

$$r = r_E + h$$
$$h = r - r_E$$
$$h = 6.9056 \times 10^6\ \text{m} - 6.38 \times 10^6\ \text{m}$$
$$h \cong 5.3 \times 10^5\ \text{m}$$

(b) Solving for T using $v = \frac{\Delta d}{\Delta t}$ to obtain $\frac{(2\pi r)}{T}$ gives $T = 5.7 \times 10^3$ s.

$$v = \frac{\Delta d}{\Delta t}$$
$$v = \frac{2\pi r}{T}$$
$$T = \frac{2\pi r}{v}$$

$$T = \frac{2\pi(6.9056 \times 10^6\ \cancel{\text{m}})}{7.6 \times 10^3\ \frac{\cancel{\text{m}}}{\text{s}}}$$
$$T \cong 5.7 \times 10^3\ \text{s}$$

31. Setting the force of gravity equal to the centripetal force and solving for v gives $v = 1.02 \times 10^3$ m/s. Solving for T using $v = \frac{\Delta d}{\Delta t}$ to obtain $\frac{(2\pi r)}{T}$ gives $T = 2.37 \times 10^6$ s.

$$G\frac{m_M \cancel{m_E}}{\cancel{r^2}} = \frac{\cancel{m_M} v^2}{\cancel{r}}$$

$$v = \sqrt{G\frac{m_E}{r}}$$

$$v = \sqrt{\left(6.67 \times 10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)\frac{5.98 \times 10^{24}\ \text{kg}}{3.84 \times 10^8\ \text{m}}}$$

$$v \cong 1.02 \times 10^3\ \frac{\text{m}}{\text{s}}$$

$$v = \frac{\Delta d}{\Delta t}$$
$$v = \frac{2\pi r}{T}$$
$$T = \frac{2\pi r}{v}$$
$$T = \frac{2\pi(3.84 \times 10^8\ \text{m})}{1.0192 \times 10^3\ \frac{\text{m}}{\text{s}}}$$
$$T \cong 2.37 \times 10^6\ \text{s}$$

32. (a) Calculating $\frac{T^2}{r^3} = k$ for each satellite gives the same value for k, or approximately $1.04 \times 10^{-15}\ \text{s}^2/\text{m}^3$, verifying that they obey Kepler's third law. Detailed calculations for Tethys appear below. Using this equation in all cases, you obtain the same value for Dione, Titan, and Iapetus. For Rhea, the value is $1.05 \times 10^{-15}\ \text{s}^2/\text{m}^3$.

$$\frac{T^2}{r^3} = k$$

$$1.888\ \cancel{\text{day}}\text{s}\left(\frac{24\ \cancel{\text{h}}}{1\ \cancel{\text{day}}}\right)\left(\frac{3600\ \text{s}}{1\ \cancel{\text{h}}}\right) = 1.6312 \times 10^5\ \text{s}$$

$$k = \frac{(1.6312 \times 10^5\ \text{s})^2}{(2.95 \times 10^8\ \text{m})^3}$$

$$k \cong 1.04 \times 10^{-15}\frac{\text{s}^2}{\text{m}^3}$$

(b) Combining Kepler's third law and Newton's law of universal gravitation and solving for m gives $m = 5.69 \times 10^{26}$ kg.

$$k = \frac{4\pi^2}{Gm_{Sa}}$$
$$m_{Sa} = \frac{4\pi^2}{Gk}$$
$$m_{Sa} = \frac{4\pi^2}{\left(6.67 \times 10^{-11}\ \frac{N \cdot \cancel{m^2}}{kg^2}\right)\left(1.04 \times 10^{-15}\ \frac{s^2}{\cancel{m^3}}\right)}$$
$$m_{Sa} \cong 5.7 \times 10^{26}\ kg$$

33. (a) Using mass = density × volume, and volume of a sphere = $\frac{4}{3}\pi r^3$, you obtain $m = 5.2 \times 10^{14}$ kg.

$$m = \rho V \qquad V = \frac{4}{3}\pi r^3$$
$$m = \rho \frac{4}{3}\pi r^3 \qquad \rho = 1.00\ \frac{g}{cm^3}\left(\frac{100\ cm}{1\ m}\right)^3\left(\frac{1\ kg}{1000\ g}\right)$$
$$\rho = 1000\ \frac{kg}{m^3}$$
$$m = \left(1000\ \frac{kg}{\cancel{m^3}}\right)\frac{4}{3}\pi(5.0 \times 10^3\ \cancel{m})^3$$
$$m \cong 5.2 \times 10^{14}\ kg$$

(b) Total mass = 5.2×10^{26} kg.

$$m_{Oort} = (1.0 \times 10^{12}\ comets)m_{comet}$$
$$m_{Oort} = (1.0 \times 10^{12}\ \cancel{comets})\left(5.2 \times 10^{14}\ \frac{kg}{\cancel{comet}}\right)$$
$$m_{Oort} = 5.2 \times 10^{26}\ kg$$

(c) The cloud's mass is 88 times larger than the mass of Earth and 3.6 times smaller than Jupiter's mass.

$$\frac{m_{Oort}}{m_E} = \frac{5.2 \times 10^{26}\ kg}{5.98 \times 10^{24}\ kg} \qquad \frac{m_{Oort}}{m_J} = \frac{5.2 \times 10^{26}\ kg}{1.90 \times 10^{27}\ kg}$$
$$\frac{m_{Oort}}{m_E} \cong 88 \qquad \frac{m_{Oort}}{m_J} \cong 0.28$$