Chapter 20

The Nucleus and Radioactivity

Practice Problem Solutions

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1. **Conceptualize the Problem**
   - The *mass defect* is the difference of the mass of the nucleus and the sum of the masses of the individual particles.

   Identify the Goal
   - The mass defect, Δm, for beryllium-8

   Identify the Variables

Known	Implied	Unknown
$m_{nucleus} = 8.003104$ u	$m_p = 1.007276$ u	N
A = 8	$m_n = 1.008665$ u	Δm
Z = 4		

Develop a Strategy	Calculations
Calculate the number of neutrons.	$N = A - Z$ $N = 8 - 4$ $N = 4$
Determine the total mass of the separate nucleons by finding the masses of the protons and neutrons and adding them together.	$m_{p(total)} = (4)(1.007276\ u)$ $m_{p(total)} = 4.029104\ u$
	$m_{n(total)} = (4)(1.008665\ u)$ $m_{n(total)} = 4.03466\ u$
	$m_{(total)} = 4.029104\ u + 4.03466\ u$ $m_{(total)} = 8.063764\ u$
Find the mass defect by subtracting the mass of the nucleus from the total nucleon mass.	$\Delta m = 8.063764\ u - 8.003104\ u$ $\Delta m = 0.06066000\ u$

   The mass defect is 0.06066000 u.

   Validate the Solution
   Mass defects are expected to be a fraction of an atomic mass unit for light nuclei, and this is the case. Note that mass defects that are not a fraction of an atomic mass unit could indicate an error in the number of protons or neutrons used. Also, mass defects that are negative could indicate that an atomic mass (which includes the mass of the electrons in the atom), instead of a nuclear mass, was used. In this text, most problems are given with nuclear masses.

2. Conceptualize the Problem

- The *mass defect* is the difference of the mass of the nucleus and the sum of the masses of the individual particles.
- The energy equivalent of the mass defect is the *binding energy* for the nucleus.

Identify the Goal

The binding energy, E, for helium-3

Identify the Variables

Known	Implied	Unknown
$m_{nucleus} = 3.014932$ u	$m_p = 1.007276$ u	N
A = 3	$m_n = 1.008665$ u	Δm
Z = 2	$c = 2.998 \times 10^8$ m/s	E

Develop a Strategy	Calculations
Calculate the number of neutrons.	$N = A - Z$ $N = 3 - 2$ $N = 1$
Determine the total mass of the separate nucleons by finding the masses of the protons and neutrons and adding them together.	$m_{p(total)} = (2)(1.007276 \text{ u})$ $m_{p(total)} = 2.014552 \text{ u}$ $m_{n(total)} = (1)(1.008665 \text{ u})$ $m_{n(total)} = 1.008665 \text{ u}$ $m_{(total)} = 2.014552 \text{ u} + 1.008665 \text{ u}$ $m_{(total)} = 3.023217 \text{ u}$
Find the mass defect by subtracting the mass of the nucleus from the total nucleon mass.	$\Delta m = 3.023217 \text{ u} - 3.014932 \text{ u}$ $\Delta m = 0.008285 \text{ u}$
Convert the mass into kilograms.	$\Delta m = (0.008285 \text{ u})(1.6605 \times 10^{-27} \text{ kg/u})$ $\Delta m = 1.375724 \times 10^{-29} \text{ kg}$
Find the energy equivalent of the mass defect.	$E = \Delta mc^2$ $E = (1.375724 \times 10^{-29} \text{ kg})(2.998 \times 10^8 \text{ m/s})^2$ $E = 1.2365 \times 10^{-12} \text{ J}$ $E = 1.237 \times 10^{-12} \text{ J}$

The binding energy is 1.237×10^{-12} J.

Validate the Solution

The binding energy per nucleon is expected to be a few MeV per nucleon (see Figure 20.4 in the text):

$$\frac{E}{A} = \frac{1.237 \times 10^{-12} \text{ J}}{1.6 \times 10^{-13} \text{ J/MeV}} \times \frac{1}{3} = 2.576 \text{ MeV, so the answer is reasonable.}$$

3. Conceptualize the Problem

- The *mass defect* is the difference of the mass of the nucleus and the sum of the masses of the individual particles.
- The energy equivalent of the mass defect is the *binding energy* for the nucleus.

Identify the Goal

The binding energy, E, for uranium-235

Identify the Variables

Known	Implied	Unknown
$m_{\text{nucleus}} = 234.9934$ u	$m_{\text{p}} = 1.007276$ u	N
A = 235	$m_{\text{n}} = 1.008665$ u	Δm
Z = 92	$c = 2.998 \times 10^8$ m/s	E

Develop a Strategy	Calculations
Calculate the number of neutrons.	$N = A - Z$ $N = 235 - 92$ $N = 143$
Determine the total mass of the separate nucleons by finding the masses of the protons and neutrons and adding them together.	$m_{\text{p(total)}} = (92)(1.007276\text{ u})$ $m_{\text{p(total)}} = 92.669392\text{ u}$ $m_{\text{n(total)}} = (143)(1.008665\text{ u})$ $m_{\text{n(total)}} = 144.239095\text{ u}$ $m_{\text{(total)}} = 92.669392\text{ u} + 144.239095\text{ u}$ $m_{\text{(total)}} = 236.908487\text{ u}$
Find the mass defect by subtracting the mass of the nucleus from the total nucleon mass.	$\Delta m = 236.908487\text{ u} - 234.9934\text{ u}$ $\Delta m = 1.915087\text{ u}$
Convert the mass into kilograms.	$\Delta m = (1.915087\text{ u})(1.6605 \times 10^{-27}\text{ kg/u})$ $\Delta m = 3.1800 \times 10^{-27}\text{ kg}$
Find the energy equivalent of the mass defect.	$E = \Delta mc^2$ $E = (3.1800 \times 10^{-27}\text{ kg})(2.998 \times 10^8\text{ m/s})^2$ $E = 2.85819 \times 10^{-10}\text{ J}$ $E \cong 2.858 \times 10^{-10}\text{ J}$

The binding energy is 2.858×10^{-10} J.

Validate the Solution

The binding energy per nucleon is expected to be several MeV per nucleon (see Figure 20.4 in the text):

$$\frac{E}{A} = \frac{2.858 \times 10^{-10}\text{ J}}{1.6 \times 10^{-13}\text{ J/MeV}} \times \frac{1}{235} = 7.602\text{ MeV, so the answer is reasonable.}$$

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4. **Conceptualize the Problem**

- The *half-life* of a radioactive isotope determines the amount of a sample at any given time.

Identify the Goal

The time interval, Δt, since the lava solidified

Identify the Variables

Known	Unknown
$m_0 = 27.4\text{ mg} \equiv N_0$	Δt
$m = 18.3\text{ mg} \equiv N$	
$T_{\frac{1}{2}} = 4.5 \times 10^9$ a	

Develop a Strategy

Write the mass decay relationship. Note, in this case, both N_0 and N are measured in mg.

Solve for the time interval.

Calculations

$$N = \left(\frac{1}{2}\right)^{\frac{\Delta t}{T_{\frac{1}{2}}}} N_0$$

$$\frac{N}{N_0} = \left(\frac{1}{2}\right)^{\frac{\Delta t}{T_{\frac{1}{2}}}}$$

$$\log\left(\frac{N}{N_0}\right) = \frac{\Delta t}{T_{\frac{1}{2}} \log\left(\frac{1}{2}\right)}$$

$$\Delta t = T_{\frac{1}{2}} \frac{\log\left(\frac{N}{N_0}\right)}{\log\left(\frac{1}{2}\right)}$$

$$\Delta t = 4.5 \times 10^9 \text{ a} \times \frac{\log\left(\frac{18.3 \text{ mg}}{27.4 \text{ mg}}\right)}{\log\left(\frac{1}{2}\right)}$$

$$\Delta t = 2.620 \times 10^9 \text{ a}$$

$$\Delta t \cong 2.6 \times 10^9 \text{ a}$$

About 2.6×10^9 years had passed since the lava solidified.

Validate the Solution

The mass of the original sample has decreased by less than a factor of two, which indicates that less than one half-life has passed. The solution is less than one half-life, so it is reasonable.

5. Conceptualize the Problem

- The *half-life* of a radioactive isotope determines the amount of a sample at any given time.

Identify the Goal

The age, Δt, of the ashes

Identify the Variables

Known	Unknown
$N_0 = 0.23$ Bq	Δt
$N = 0.15$ Bq	
$T_{\frac{1}{2}} = 5730$ a	

Develop a Strategy

Write the mass decay relationship. Note, in this case, both N_0 and N are measured in Bq.

Solve for the time interval.

Calculations

$$N = \left(\frac{1}{2}\right)^{\frac{\Delta t}{T_{\frac{1}{2}}}} N_0$$

$$\frac{N}{N_0} = \left(\frac{1}{2}\right)^{\frac{\Delta t}{T_{\frac{1}{2}}}}$$

$$\log\left(\frac{N}{N_0}\right) = \frac{\Delta t}{T_{\frac{1}{2}}} \log\left(\frac{1}{2}\right)$$

$$\Delta t = T_{\frac{1}{2}} \frac{\log\left(\frac{N}{N_0}\right)}{\log\left(\frac{1}{2}\right)}$$

$$\Delta t = 5730 \text{ a} \times \frac{\log\left(\frac{0.15 \text{ Bq}}{0.23 \text{ Bq}}\right)}{\log\left(\frac{1}{2}\right)}$$

$$\Delta t = 3533 \text{ a}$$

$$\Delta t \cong 3.5 \times 10^3 \text{ a}$$

The ashes are about 3.5×10^3 years old.

Validate the Solution
The original activity has decreased by less than a factor of two, which indicates that less than one half-life has passed. The solution is less than one half-life, so it is reasonable.

6. **Conceptualize the Problem**
- The *half-life* of a radioactive isotope determines the amount of a sample at any given time.

Identify the Goal
The amount of I-128, N, remaining after 12.0 h

Identify the Variables

Known	Unknown
$\Delta t = 12.0$ h	N
$N_0 = 40.0$ mg	
$T_{\frac{1}{2}} = 24.99$ min $= 0.4165$ h	

Develop a Strategy	Calculations
Write the mass decay relationship.	$N = \left(\frac{1}{2}\right)^{\frac{\Delta t}{T_{\frac{1}{2}}}} N_0$
Substitute values and solve.	$N = \left(\frac{1}{2}\right)^{\frac{12.0\text{ h}}{0.4165\text{ h}}} (40.0\text{ mg})$
	$N = 8.49 \times 10^{-8}$ mg
After 12.0 h, 8.49×10^{-8} mg of the sample remain.	

Validate the Solution
The time interval, 12.0 h, is more than 24 half-lives, so it is expected that most of the original amount of I-128 will have decayed. The answer seems reasonable.

Chapter 20 Review

Answers to Problems for Understanding

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12. **(a)** Calcium: 20 p, 18 e, 20 n
(b) Iron: 26 p, 26 e, 30 n
(c) Chlorine: 17 p, 18 e, 18 n

13. **(a)** Carbon $^{12}_{6}$C has six protons and six neutrons in the nucleus.
$\Delta m = Zm_p + Nm_n - (m_{C-12} - Zm_e)$
$\Delta m = 6(1.007\ 276\text{ u}) + 6(1.008\ 665\text{ u}) - [12\text{ u} - 6(0.000\ 549\text{ u})]$
$\Delta m = 0.098\ 94\text{ u}$
$\Delta E = \Delta mc^2$
$\Delta E = (0.098\ 94\ \cancel{\text{u}})(1.6605 \times 10^{-27}\ \frac{\text{kg}}{\cancel{\text{u}}})(2.9979 \times 10^8\ \frac{\text{m}}{\text{s}})^2$
$\Delta E = 1.4765 \times 10^{-11}$ J or 9.217×10^7 eV
(b) Cesium $^{133}_{55}$CS has 55 protons and 78 neutrons.
$\Delta m = Zm_p + Nm_n - (m_{Cs-133} - Zm_e)$
$\Delta m = 55(1.007\ 276\text{ u}) + 78(1.008\ 665\text{ u}) - [132.905\text{ u} - 55(0.000\ 549\text{ u})]$
$\Delta m = 1.201\ 245\text{ u}$

$$\Delta E = \Delta mc^2$$
$$\Delta E = (1.201\ 245\ \cancel{u})\left(1.6605 \times 10^{-27}\ \frac{\text{kg}}{\cancel{u}}\right)\left(2.9979 \times 10^{8}\ \frac{\text{m}}{\text{s}}\right)^2$$
$$\Delta E = 1.7927 \times 10^{-01}\ \text{J or } 1.119 \times 10^{9}\ \text{eV}$$

14. ${}^{230}_{90}\text{Th} \rightarrow {}^{4}_{2}\text{He} + {}^{226}_{88}\text{Ra}$

15. (a) $\left(\frac{1}{2}\right)^2 = \frac{1}{4}$ **(b)** $\left(\frac{1}{2}\right)^4 = \frac{1}{16}$ **(c)** $\left(\frac{1}{2}\right)^{12} = \frac{1}{4096}$

16. (a) Mass defect is mass of radium nucleus – mass of radon nucleus – mass of alpha particle.

$$\Delta m = 225.977\ 09\ \text{u} - 221.970\ 356\ \text{u} - 4.002\ 602\ \text{u}$$
$$\Delta m = 0.004\ 132\ \text{u}$$
$$E = \Delta mc^2$$
$$E = (0.004\ 132\ \text{u})\left(1.6605 \times 10^{-27}\ \frac{\text{kg}}{\text{u}}\right)\left(3.0 \times 10^{8}\ \frac{\text{m}}{\text{s}}\right)^2$$
$$E = 6.175 \times 10^{-13}\ \text{J or } 3.85\ \text{MeV}$$

(b) You have to conserve momentum.

$$m_{\text{radon}}v_{\text{radon}} = m_{\text{helium}}v_{\text{helium}}$$
$$(221.970\ 356\ \cancel{u})\left(1.6605 \times 10^{-27}\ \frac{\text{kg}}{\cancel{u}}\right)v_{\text{radon}} = (4.002\ 602\ \cancel{u})\left(1.6605 \times 10^{-27}\ \frac{\text{kg}}{\cancel{u}}\right)v_{\text{helium}}$$
$$v_{\text{helium}} = 55.46v_{\text{radon}}$$

You have to conserve energy.

$$\frac{1}{2}m_{\text{radon}}v_{\text{radon}}^2 + \frac{1}{2}m_{\text{helium}}v_{\text{helium}}^2 = 6.175 \times 10^{-13}\ \text{J}$$
$$\frac{1}{2}(221.970\ 356\ \cancel{u})\left(1.6605 \times 10^{-27}\ \frac{\text{kg}}{\cancel{u}}\right)v_{\text{radon}}^2 + \frac{1}{2}(4.002\ 602\ \cancel{u})\left(1.6605 \times 10^{-27}\ \frac{\text{kg}}{\cancel{u}}\right)v_{\text{helium}}^2 = 6.175 \times 10^{-13}\ \text{J}$$
$$1.8429 \times 10^{-25}v_{\text{radon}}^2 + 3.3232 \times 10^{-27}v_{\text{helium}}^2 = 6.175 \times 10^{-13}\ \text{J}$$

Substitute for v_{helium}.

$$1.8429 \times 10^{-25}v_{\text{radon}}^2 + 3.3232 \times 10^{-27}(55.46v_{\text{radon}})^2 = 6.175 \times 10^{-13}\ \text{J}$$
$$v_{\text{radon}} = 2.436 \times 10^{5}\ \frac{\text{m}}{\text{s}}$$
$$v_{\text{helium}} = 55.46v_{\text{radon}}$$
$$v_{\text{helium}} = 55.46\left(2.436 \times 10^{5}\ \frac{\text{m}}{\text{s}}\right)$$
$$v_{\text{helium}} = 1.35 \times 10^{7}\ \frac{\text{m}}{\text{s}}$$

(c) $$\frac{\frac{1}{2}m_{\text{alpha}}v_{\text{alpha}}^2}{E_{\text{total}}} \times 100 = \frac{\frac{1}{2}(4.002\ 602\ \cancel{u})\left(1.6605 \times 10^{-27}\ \frac{\text{kg}}{\cancel{u}}\right)\left(1.35 \times 10^{7}\ \frac{\text{m}}{\text{s}}\right)^2}{6.175 \times 10^{-13}\ \text{J}} \times 100$$
$$= 98.1\%$$

17. $$N = N_o\left(\frac{1}{2}\right)^{\frac{\Delta t}{T_{\frac{1}{2}}}}$$
$$N = (0.25\ \text{g})\left(\frac{1}{2}\right)^{\frac{21\ \text{days}}{1\ \text{day}}}$$
$$N = 1.19 \times 10^{-7}\ \text{g}$$

18.

$$N = N_o\left(\frac{1}{2}\right)^{\frac{\Delta t}{T_{\frac{1}{2}}}}$$

$$10 \times 10^{-6}\ \text{g} = \left(125 \times 10^{-3}\ \text{g}\right)\left(\frac{1}{2}\right)^{\frac{\Delta t}{3.16\ \text{min}}}$$

$$\frac{10 \times 10^{-6}\ \text{g}}{125 \times 10^{-3}\ \text{g}} = \left(\frac{1}{2}\right)^{\frac{\Delta t}{3.16\ \text{min}}}$$

$$\frac{\log 8.0 \times 10^{-5}}{\log 0.5} = \frac{\Delta t}{3.16\ \text{min}}$$

$$\Delta t = 43\ \text{min}$$

19.

$$N = N_o\left(\frac{1}{2}\right)^{\frac{\Delta t}{T_{\frac{1}{2}}}}$$

$$5.7 \times 10^{-2}\ \text{Bq} = (0.23\ \text{Bq})\left(\frac{1}{2}\right)^{\frac{\Delta t}{5.73 \times 10^3\ \text{a}}}$$

$$0.247\ 83 = \left(\frac{1}{2}\right)^{\frac{\Delta t}{5.73 \times 10^3\ \text{a}}}$$

$$\frac{\log 0.247\ 83}{\log 0.5} = \frac{\Delta t}{5.73 \times 10^3\ \text{a}}$$

$$\Delta t = 1.15 \times 10^4\ \text{a}$$

$$\Delta t \cong 1.2 \times 10^4\ \text{a}$$

20. (a) Ten minutes is two half-lives, so one quarter of the original sample would remain.

$$\frac{800\ \text{atoms}}{4} = 200\ \text{atoms}$$

(b) After 10 min, $800 - 200 = 600$ atoms of daughter nuclei that must have been produced.

(c) There must be $\frac{1}{2^5} = \frac{1}{32}$ of the parent nuclei left, because 25 min is 5 half-lives.

$$\frac{800\ \text{atoms}}{32} = 25\ \text{atoms}$$

(d) After 25 min, $800 - 25 = 775$ atoms of daughter nuclei that must have been produced.

(e) $N = N_o\left(\frac{1}{2}\right)^{\frac{\Delta t}{T_{\frac{1}{2}}}}$

$$N = (800\ \text{atoms})\left(\frac{1}{2}\right)^{\frac{\Delta t}{5\ \text{min}}}$$

21. (a) $N = N_o\left(\frac{1}{2}\right)^{\frac{\Delta t}{T_{\frac{1}{2}}}}$

$$N_{Pb} = N_{U0} - N_U$$

$$N_{Pb} = N_{U0} - \left(\frac{1}{2}\right)^{\frac{\Delta t}{T_{\frac{1}{2}}}} N_{U0}$$

$$\frac{N_U}{N_{Pb}} = \frac{\left(\frac{1}{2}\right)^{\frac{\Delta t}{T_{\frac{1}{2}}}} N_{U0}}{N_{U0} - \left(\frac{1}{2}\right)^{\frac{\Delta t}{T_{\frac{1}{2}}}} N_{U0}}$$

$$\frac{N_U}{N_{Pb}} = \frac{\left(\frac{1}{2}\right)^{\frac{\Delta t}{T_{\frac{1}{2}}}}}{1 - \left(\frac{1}{2}\right)^{\frac{\Delta t}{T_{\frac{1}{2}}}}}$$

(b) For a ratio of 1.08:1

$$\frac{N_U}{N_{Pb}} = \frac{\left(\frac{1}{2}\right)^{\frac{\Delta t}{T_{\frac{1}{2}}}}}{1 - \left(\frac{1}{2}\right)^{\frac{\Delta t}{T_{\frac{1}{2}}}}}$$

$$1.08 = \frac{\left(\frac{1}{2}\right)^{\frac{\Delta t}{4.5 \times 10^9 \text{ a}}}}{1 - \left(\frac{1}{2}\right)^{\frac{\Delta t}{4.5 \times 10^9}}}$$

$$1.08 - 1.08\left(\frac{1}{2}\right)^{\frac{\Delta t}{4.5 \times 10^9 \text{ a}}} = \left(\frac{1}{2}\right)^{\frac{\Delta t}{4.5 \times 10^9 \text{ a}}}$$

$$1.08 = 2.08\left(\frac{1}{2}\right)^{\frac{\Delta t}{4.5 \times 10^9 \text{ a}}}$$

$$0.519 = \left(\frac{1}{2}\right)^{\frac{\Delta t}{4.5 \times 10^9 \text{ a}}}$$

$$\log 0.519 = \frac{\Delta t}{4.5 \times 10^9 \text{ a}} \log\left(\frac{1}{2}\right)$$

$$\Delta t = 4.26 \times 10^9 \text{ a}$$

$$\Delta t \cong 4.3 \times 10^9 \text{ a}$$

Similarly, for a ratio of 1.22:1, $\Delta t = 3.9 \times 10^9$ a and for a ratio of 1.75:1, $\Delta t = 2.9 \times 10^9$ a

(c) Since the ratios and therefore the ages differ, the rocks must not have solidified at the same time.

(d) If the ratio is less than one, more than half of the uranium has decayed. Therefore, more than a half-life (or more than 4.5×10^9 years) has passed since the rock solidified.