1. (a) Frame the Problem
- Make a sketch of the vector.
- The angle is between 0˚ and 90˚ so it is in the first quadrant. Therefore, both the x-component and the y-component will be positive.
- Use trigonometric functions to find components of the vector.

Identify the Goal
The components $\Delta d_x$, $\Delta d_y$ of vector $\Delta \vec{d}$

Variables and Constants

<table>
<thead>
<tr>
<th>Known</th>
<th>Unknown</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta \vec{d}$</td>
<td>$\Delta d_x$</td>
<td>$\Delta d_y$</td>
</tr>
<tr>
<td>$\theta = 75^\circ$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Strategy
Draw the vector with its tail at the origin of an $x$-$y$-coordinate system.

Draw lines from the tip of the vector to each axis, so that they are parallel to the axes.

Calculate the components.
$\Delta d_x = |\Delta \vec{d}| \cos \theta$
$\Delta d_x = 16 \text{ m} \cos 75^\circ$
$\Delta d_x = 4.14 \text{ m}$

$\Delta d_y = |\Delta \vec{d}| \sin \theta$
$\Delta d_y = 16 \text{ m} \sin 75^\circ$
$\Delta d_y = 15.45 \text{ m}$

Determine the signs of the components
The $x$-component lies on the positive $x$-axis so it is positive. The $y$-component lies on the positive $y$-axis so it is positive.

(a) The $x$-component of the vector is $+4.1 \text{ m}$ and the $y$-component of the vector is $+15 \text{ m}$.

Validate
Use the Pythagorean theorem to check your answers.
$|\Delta \vec{d}|^2 = \Delta d_x^2 + \Delta d_y^2$
$|\Delta \vec{d}|^2 = (4.14 \text{ m})^2 + (15.45 \text{ m})^2$
$|\Delta \vec{d}|^2 = 255.842$
$|\Delta \vec{d}| = 16 \text{ m}$

The value agrees with original vector.
1. **(b) Frame the Problem**
   - Make a sketch of the vector.
   - The angle is between 90˚ and 180˚ so it is in the second quadrant. Therefore, the $x$-component will be negative and the $y$-component will be positive.
   - Use trigonometric functions to find components of the vector.

**Identify the Goal**
The components $a_x$, $a_y$ of vector $\vec{a}$.

**Variables and Constants**

<table>
<thead>
<tr>
<th>Known</th>
<th>Unknown</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\vec{a}$ = 8.1 m/s²</td>
<td>$a_x$</td>
</tr>
<tr>
<td>$\theta$ = 145˚</td>
<td>$a_y$</td>
</tr>
</tbody>
</table>

**Strategy**
Draw the vector with its tail at the origin of an $x$-$y$-coordinate system.

Identify the angle with the closest $x$-axis and label it $\theta_R$.

Draw lines from the tip of the vector to each axis, so that they are parallel to the axes.

Calculate the components.

- $a_x = |\vec{a}| \cos \theta_R$
- $a_x = 8.1 \text{ m/s} \cos 35˚$
- $a_x = 6.635 \text{ m/s}^2$
- $a_y = |\vec{a}| \sin \theta_R$
- $a_y = 8.1 \text{ m/s} \sin 35˚$
- $a_y = 4.646 \text{ m/s}^2$

Determine the signs of the components

- The $x$-component lies on the negative $x$-axis so it is negative. The $y$-component lies on the positive $y$-axis so it is positive.

**Validate**
Use the Pythagorean theorem to check your answers.

- $|\vec{a}|^2 = a_x^2 + a_y^2$
- $|\vec{a}|^2 = (6.635 \text{ m/s})^2 + (4.646 \text{ m/s})^2$
- $|\vec{a}|^2 = 65.6085 \text{ (m/s)}^2$
- $|\vec{a}| = 8.1 \text{ m/s}$

The value agrees with the magnitude of the original vector.

1. **(c) Frame the Problem**
   - Make a sketch of the vector.
   - The angle is between 180˚ and 270˚ so it is in the third quadrant. Therefore, the $x$-component will be negative and the $y$-component will be negative.
   - Use trigonometric functions to find components of the vector.

**Identify the Goal**
The components $v_x$, $v_y$ of vector $\vec{v}$.
Variables and Constants

<table>
<thead>
<tr>
<th>Known</th>
<th>Unknown</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\vec{v} = 16.0 \text{ m/s}$</td>
<td>$v_x$</td>
</tr>
<tr>
<td>$\theta = 225^\circ$</td>
<td>$v_y$</td>
</tr>
</tbody>
</table>

**Strategy**

Draw the vector with its tail at the origin of an $x$-$y$-coordinate system.

Identify the angle with the closest $x$-axis and label it $\theta_R$.

Draw lines from the tip of the vector to each axis, so that they are parallel to the axes.

Calculate the components.

$v_x = |\vec{v}| \cos \theta_R$
$v_x = 16.0 \text{ m/s} \cos 45^\circ$
$v_x = 11.314 \text{ m/s}$

$v_y = |\vec{v}| \sin \theta_R$
$v_y = 16.0 \text{ m/s} \sin 45^\circ$
$v_y = 11.314 \text{ m/s}$

Determine the signs of the components

The $x$-component lies on the negative $x$-axis so it is negative. The $y$-component lies on the negative $y$-axis so it is negative.

(c) The $x$-component of the vector is $-11.3 \text{ m/s}$ and the $y$-component of the vector is $-11.3 \text{ m/s}$.

**Validate**

Use the Pythagorean theorem to check your answers.

$|\vec{v}|^2 = v_x^2 + v_y^2$

$|\vec{v}|^2 = (11.314 \text{ m/s})^2 + (11.314 \text{ m/s})^2$

$|\vec{v}|^2 = 256.013 \text{(m/s)}^2$

$|\vec{v}| = 16.0 \text{ m/s}$

The value agrees with the magnitude of the original vector.

2. (a) **Frame the Problem**

- Make a sketch of the vector.
- The vector is described in compass directions.
- Convert to an $x$-$y$-coordinate system. Let $+y$ be north, and $-y$ south. East will become $+x$ and west $-x$.
- You will need to find the angle with the closest $x$-axis.

**Identify the Goal**

The components $\Delta d_x$, $\Delta d_y$ of vector $\vec{\Delta d} = 20.0 \text{ km}[N20.0^\circ E]$.

**Variables and Constants**

<table>
<thead>
<tr>
<th>Known</th>
<th>Unknown</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta d = 20.0 \text{ km}[N20.0^\circ E]$</td>
<td>$\Delta d_x$</td>
</tr>
<tr>
<td>$\Delta d_y$</td>
<td>$\theta$</td>
</tr>
</tbody>
</table>
Strategy Calculations
Draw an \( x-y \)-coordinate system and indicate that the axes also represent compass directions.

Draw the vector with its tail at the origin.
Identify the angle, \( \theta \)
\[ \theta = 90^\circ - 20.0^\circ \]
\[ \theta = 70^\circ \]

Use the 20.0° made by the vector to the \( y \)-axis to find the angle the vector makes with the \( x \)-axis.

Draw lines from the tip of the vector to each axis, so that they are parallel to the axes.

Calculate the components.
\[ \Delta d_x = \left| \Delta d \right| \cos \theta \]
\[ \Delta d_x = 20.0 \text{ km} \cos 70^\circ \]
\[ \Delta d_x = 6.84 \text{ km} \]

\[ \Delta d_y = \left| \Delta d \right| \sin \theta \]
\[ \Delta d_y = 20.0 \text{ km} \sin 70^\circ \]
\[ \Delta d_y = 18.794 \text{ km} \]

Determine the signs of the components
The \( x \)-component lies on the positive \( x \)-axis so it is positive. The \( y \)-component lies on the positive \( y \)-axis so it is positive.

(a) The \( x \)-component of the vector is \(+6.84 \text{ km}\) and the \( y \)-component of the vector is \(+18.8 \text{ km}\).

Validate
Use the Pythagorean theorem to check your answers.
\[ \left| \Delta d \right|^2 = \Delta d_x^2 + \Delta d_y^2 \]
\[ \left| \Delta d \right|^2 = (6.840 \text{ km})^2 + (18.794 \text{ km})^2 \]
\[ \left| \Delta d \right|^2 = 400.000 \]
\[ \left| \Delta d \right| = 20.0 \text{ km} \]

The value agrees with original vector.

2. (b) Frame the Problem
- Make a sketch of the vector.
- The vector is described in compass directions.
- Convert to an \( x-y \)-coordinate system. Let \(+y\) be north, and \(-y\) south. East will become \(+x\) and west will be \(-x\).
- You will need to find the angle with the closest \( x \)-axis.

Identify the Goal
The components \( v_x, v_y \) of vector \( \vec{v} = 3.0 \text{ m/s}\text{[E30.0°S]} \)

Variables and Constants
\begin{align*}
\text{Known} & & \text{Unknown} \\
\vec{v} = 16.0 \text{ m/s}[E30.0°S] & & v_x \\
& & v_y \\
& & \theta
\end{align*}
Strategy

Draw an \( x-y \)-coordinate system and indicate that the axes also represent compass directions. Draw the vector with its tail at the origin. Identify the angle, \( \theta \).

The vector makes an angle \( \theta = 30.0^\circ \) with the \( x \)-axis.

Draw lines from the tip of the vector to each axis, so that they are parallel to the axes.

Calculate the components.

\[
\begin{align*}
v_x &= |\vec{v}| \cos \theta \\
v_x &= 3.0 \, \text{m/s} \cos 30^\circ \\
v_x &= 2.598 \, \text{m/s} \\
v_y &= |\vec{v}| \sin \theta \\
v_y &= 3.0 \, \text{m/s} \sin 30^\circ \\
v_y &= 1.50 \, \text{m/s}
\end{align*}
\]

Determine the signs of the components

The \( x \)-component lies on the positive \( x \)-axis so it is positive. The \( y \)-component lies on the negative \( y \)-axis so it is negative.

\[\text{(b)} \quad \text{The } x\text{-component of the vector is } -2.6 \, \text{m/s and the } y\text{-component of the vector is } -1.5 \, \text{m/s.}\]

Validate

Use the Pythagorean theorem to check your answers.

\[
|\vec{v}|^2 = v_x^2 + v_y^2
\]

\[
|\vec{v}|^2 = (2.598 \, \text{m/s})^2 + (1.50 \, \text{m/s})^2
\]

\[
|\vec{v}|^2 = 8.9996 (\text{m/s})^2
\]

\[
|\vec{v}| = 3.0 \, \text{m/s}
\]

The value agrees with the magnitude of the original vector.

2. (c) Frame the Problem

- Make a sketch of the vector.
- The vector is described in compass directions.
- Convert to an \( x-y \)-coordinate system. Let \( +y \) be north, and \( -y \) south. East will become \( +x \) and west will be \( -x \).
- You will need to find the angle with the closest \( x \)-axis.

Identify the Goal

The components \( v_x, v_y \) of vector \( \vec{v} \) = 6.8 m/s[W70.0°N]

Variables and Constants

\[
\begin{array}{ll}
\text{Known} & \text{Unknown} \\
\vec{v} = 6.8 \, \text{m/s} [W70.0^\circ N] & v_x \\
& v_y \\
& \theta
\end{array}
\]
Strategy

Draw an $x$-$y$-coordinate system and indicate that the axes also represent compass directions. Draw the vector with its tail at the origin. Identify the angle, $\theta$. The vector makes an angle $\theta = 70.0^\circ$ with the $x$-axis.

Draw lines from the tip of the vector to each axis, so that they are parallel to the axes.

Calculate the components.

\[
\begin{align*}
\nu_x &= |\vec{v}| \cos \theta \\
\nu_x &= 6.8 \text{ m/s} \cos 70^\circ \\
\nu_x &= 2.326 \text{ m/s} \\
\nu_y &= |\vec{v}| \sin \theta \\
\nu_y &= 6.8 \text{ m/s} \sin 70^\circ \\
\nu_y &= 6.390 \text{ m/s}
\end{align*}
\]

Determine the signs of the components

The $x$-component lies on the negative $x$-axis so it is negative. The $y$-component lies on the positive $y$-axis so it is positive.

(c) The $x$-component of the vector is $-2.3 \text{ m/s}$ and the $y$-component of the vector is $+6.4 \text{ m/s}$.

Validate

Use the Pythagorean theorem to check your answers.

\[
|\vec{v}|^2 = \nu_x^2 + \nu_y^2
\]

\[
|\vec{v}|^2 = (2.326 \text{ m/s})^2 + (6.390 \text{ m/s})^2
\]

\[
|\vec{v}| = 6.8 \text{ m/s}
\]

The value agrees with the magnitude of the original vector.

3. Frame the Problem

- Make a sketch of the situation.
- The position of the balloon is described in compass directions.
- Resolving the vector into components will allow you to determine how the pickup vehicle should reach the balloon.
- Convert to an $x$-$y$-coordinate system. Let $+y$ be north, and $-y$ south. East will become $+x$ and west will be $-x$.
- You will need to find the angle with the closest $x$-axis.

Identify the Goal

The components $\Delta d_x$, $\Delta d_y$ of vector $\Delta \vec{d} = 60.0 \text{ km}[E60.0^\circ \text{N}]$.

Variables and Constants

<table>
<thead>
<tr>
<th>Known</th>
<th>Unknown</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta \vec{d} = 60.0 \text{ km}[E60.0^\circ \text{N}]$</td>
<td>$\Delta d_x$</td>
</tr>
<tr>
<td>$\Delta d_y$</td>
<td>$\theta$</td>
</tr>
</tbody>
</table>
### Strategy

Draw an $x$-$y$-coordinate system and indicate that the axes also represent compass directions. Draw the vector with its tail at the origin. Identify the angle, $\theta$.

The vector makes an angle of $\theta = 60.0^\circ$ with the positive $x$-axis.

Draw lines from the tip of the vector to each axis, so that they are parallel to the axes.

Calculate the components.

\[
\Delta d_x = |\Delta \vec{d}| \cos \theta \\
\Delta d_x = 60.0 \text{ km} \cos 60^\circ \\
\Delta d_x = 30.0 \text{ km} \\
\Delta d_y = |\Delta \vec{d}| \sin \theta \\
\Delta d_y = 60.0 \text{ km} \sin 60^\circ \\
\Delta d_y = 51.962 \text{ km}
\]

Determine the signs of the components

The $x$-component lies on the positive $x$-axis so it is positive. The $y$-component lies on the positive $y$-axis so it is positive.

The pickup vehicle will have to drive $3.0 \times 10^1 \text{ km}$ east and then $52 \text{ km}$ north to pick up the balloon.

### Calculations

- \( \Delta d_x = |\Delta \vec{d}| \cos \theta \)
- \( \Delta d_x = 60.0 \text{ km} \cos 60^\circ \)
- \( \Delta d_x = 30.0 \text{ km} \)
- \( \Delta d_y = |\Delta \vec{d}| \sin \theta \)
- \( \Delta d_y = 60.0 \text{ km} \sin 60^\circ \)
- \( \Delta d_y = 51.962 \text{ km} \)

- The $x$-component lies on the positive $x$-axis so it is positive. The $y$-component lies on the positive $y$-axis so it is positive.

### Practice Problem Solutions

**Student Textbook page 463**

**4. Frame the Problem**

- Make a diagram of the problem.
- The hiker’s trip is in three segments which are given in terms of displacements and compass directions. You can convert the compass directions to the $x$-$y$-coordinate system.
- The total displacement is the vector sum of the three displacement vectors.
- The components of each displacement can be determined and summed to get the components of the resultant vector.

**Identify the Goal**

(a) The displacement of the boat, \( \Delta \vec{d}_{\text{total}} \)

(b) The direction, $\theta$, the boat would need in order to head straight home
Variables and Constants

<table>
<thead>
<tr>
<th>Known</th>
<th>Unknown</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta \vec{d}_A = 2.7 \text{ km}[S]$</td>
<td>$\Delta \vec{d}_{\text{total}}$</td>
</tr>
<tr>
<td>$\Delta \vec{d}_B = 3.4 \text{ km}[S26^\circ\text{E}]$</td>
<td>$\Delta d_{Ax}$ $\Delta d_{Ay}$</td>
</tr>
<tr>
<td>$\vec{d}_C = 1.9 \text{ km}[E12^\circ\text{N}]$</td>
<td>$\Delta d_{Bx}$ $\Delta d_{By}$</td>
</tr>
<tr>
<td></td>
<td>$\Delta d_{Cx}$ $\Delta d_{Cy}$</td>
</tr>
<tr>
<td>$\theta_A$ $\theta_B$ $\theta_C$</td>
<td></td>
</tr>
</tbody>
</table>

Strategy

Draw the first displacement on an $x$-$y$-coordinate system with $+y$ corresponding to north.

Find the angle the vector makes with the $x$-axis.

Calculate the $x$- and $y$-components of displacement $A$ and determine the signs of the components.

Draw displacement $B$ and calculate its $x$- and $y$-components.

Determine the angle the vector makes with the $x$-axis, $\theta_B$.

Calculate the $x$- and $y$-components of displacement $B$.

Determine the signs of the components.

Draw displacement $C$ and calculate its $x$- and $y$-components.

Determine the angle the vector makes with the $x$-axis, $\theta_C$.

Calculate the $x$- and $y$-components of displacement $B$.

Determine the signs of the components.

The components of the resultant vector can be summed individually.

Calculations

Because displacement $A$ lies along the negative $y$-axis, the angle $\theta_A = 0^\circ$.

Thus, the $x$-component will be zero, and $y$-component will be negative.

$\Delta d_{Ax} = 0$
$\Delta d_{Ay} = -2.7 \text{ km}$
$\theta_B = 90^\circ - 26^\circ$
$\theta_B = 64^\circ$

$\Delta d_{Bx} = |\Delta \vec{d}_B| \cos \theta$
$\Delta d_{Bx} = 3.4 \text{ km} \cos 64^\circ$
$\Delta d_{Bx} = 1.490 \text{ km}$
$\Delta d_{By} = |\Delta \vec{d}_B| \sin \theta$
$\Delta d_{By} = 3.4 \text{ km} \sin 64^\circ$
$\Delta d_{By} = 3.056 \text{ km}$

The vector lies in the 4th quadrant, so the $x$-component is positive and the $y$-component is negative.

$\theta_C = 12^\circ$

$\Delta d_{Cx} = |\Delta \vec{d}_C| \cos \theta$
$\Delta d_{Cx} = 1.9 \text{ km} \cos 12^\circ$
$\Delta d_{Cx} = 1.858 \text{ km}$
$\Delta d_{Cy} = |\Delta \vec{d}_C| \sin \theta$
$\Delta d_{Cy} = 1.9 \text{ km} \sin 12^\circ$
$\Delta d_{Cy} = 0.395 \text{ km}$

The vector lies in the first quadrant, so the $x$- and $y$-components are positive.

$\Delta d_{\text{total}x} = \Delta d_{Ax} + \Delta d_{Bx} + \Delta d_{Cx}$
$\Delta d_{\text{total}x} = 0 + 1.490 \text{ km} + 1.858 \text{ km}$
$\Delta d_{\text{total}x} = 3.348 \text{ km}$
$\Delta d_{\text{total}y} = \Delta d_{Ay} + \Delta d_{By} + \Delta d_{Cy}$
$\Delta d_{\text{total}y} = -2.7 - 3.056 \text{ km} + 0.395 \text{ km}$
$\Delta d_{\text{total}y} = -5.361 \text{ km}$
Use the Pythagorean theorem to determine the magnitude of the resultant displacement.

\[(\Delta d_{\text{total}})^2 = (\Delta d_{\text{total},x})^2 + (\Delta d_{\text{total},y})^2\]

\[(\Delta d_{\text{total}})^2 = (3.348 \text{ km})^2 + (-5.361 \text{ km})^2\]

\[\Delta d_{\text{total}} = 6.320 \text{ km}\]

Use the components to determine the direction of the resultant displacement.

\[\tan \theta = \frac{-5.361 \text{ km}}{3.348 \text{ km}}\]

\[\tan \theta = -1.6012\]

\[\theta = \tan^{-1} 1.6012\]

\[\theta = -58.01^\circ\]

(a) The displacement is 6.3 km[E58°S].

(b) To head straight back, the boat should head in the direction [W58°N]

Validate

In each case, the units cancelled to give the correct units for the desired quantity. The displacement, 6.3 km, is less than the total distance travelled (2.7 km + 3.4 km + 1.9 km), as it should be. The solution can be checked using a graphical method.

5. Frame the Problem
- Make a diagram of the problem.
- The jet-ski wants to travel to an island and must aim further upriver to compensate for the current which is flowing to the east.
- The velocity of the jet-ski relative to the shore, \(\vec{v}_s\), is the vector sum of the velocity of the river relative to the shore, \(\vec{v}_r\), and the velocity of the jet-ski relative to the river, \(\vec{v}_{jr}\). The magnitude of the velocity of the jet-ski relative to the river, \(\vec{v}_{jr}\), is unknown, but its direction is known.
- In other words, the resultant velocity is the sum of the jet ski’s heading, and the river velocity. The magnitude of the jet-ski’s heading is given, but not its direction. Conversely, the direction of the resultant velocity is given, but not its magnitude.
- The sine law can be used.

Identify the Goal
(a) The direction the jet-ski must head in order to travel to the island.

(b) The time it will take him to reach the island, \(\Delta t\).

Variables and Constants

<table>
<thead>
<tr>
<th>Known</th>
<th>Implied</th>
<th>Unknown</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\vec{v}_s) = 40.0 km/h[(\theta)]</td>
<td>(\theta + \phi = 20^\circ)</td>
<td>(\theta)</td>
</tr>
<tr>
<td>(\vec{v}_r) = 6.0 km/h[E]</td>
<td></td>
<td>(\phi)</td>
</tr>
<tr>
<td>(\Delta \vec{d}) = 5.0 km/h[W20.0°S]</td>
<td>(\vec{v}_{jr})</td>
<td>(\Delta t)</td>
</tr>
</tbody>
</table>

\[\theta = 20^\circ\]

\[\vec{v}_s = 40.0 \text{ km/h}\]

\[\vec{v}_r = 6.0 \text{ km/h[E]}\]

\[\phi = 160^\circ\]

\[\phi = \theta = 20^\circ\]
Develop a Strategy

Determine how the vectors add together.

Note that it would be difficult to use components here because we know the magnitude of \( \vec{v}_p \) but not its direction (\( \theta \)) and we know the direction of \( \vec{v}_s \) but not its magnitude.

From the figure, it is evident that the sine law can be used to solve for the unknown velocity, \( \vec{v}_j \), and the unknown direction, \( \theta \).

Note that \( \theta + \varphi = 20^\circ \)

Solve for \( \theta \).

(a) He should head the jet-ski in the direction [W17.1°S].

Apply the sine law again to find the unknown velocity, \( \vec{v}_j \).

Find the time using the definition for the average velocity.

(b) It will take him 8.7 min to reach the island.
Validate the Solution

Once the direction of the heading (i.e. the velocity of the jet-ski relative to the water) is found, vector addition can be used to determine the velocity of the jet-ski relative to the shore (or the resultant vector). The solution can be found from components:

\[ x: \quad v_{jx} = v_{jrx} + v_{rsx} \]

or,

\[ v_{jx} \cos 20.0^\circ = v_{jx} \cos \theta + v_{rsx} \] (1)

\[ y: \quad v_{jy} = v_{jry} \]

or,

\[ v_{jy} \sin 20.0^\circ = v_{jy} \sin \theta \] (2)

Either equation, (1) or (2) can be solved:

From equation (1):

\[ v_{jx} \cos 20.0^\circ = v_{jx} \cos \theta + v_{rsx} \]

\[ v_{jx} = \frac{v_{jx} \cos \theta + v_{rsx}}{\cos 20.0^\circ} = \frac{(40 \text{ km} / \text{h}) \cos 17.06 + (6.0 \text{ km} / \text{h})}{\cos 20.0^\circ} \]

\[ v_{jx} = 34.31 \text{ km} / \text{h} \]

or, from equation (2):

\[ v_{jx} \sin 20.0^\circ = v_{jx} \sin \theta \]

\[ v_{jx} = \frac{v_{jx} \sin \theta}{\sin 20.0^\circ} = \frac{(40.0 \text{ km} / \text{h}) \sin 17.06}{\sin 20.0^\circ} \]

\[ v_{jx} = 34.31 \text{ km} / \text{h} \]

The magnitude of the velocity calculated here agrees with that calculated from the sine law above.

6. Frame the Problem

- Make a sketch of the motion in the problem.
- Vector addition will be used.
- The vector sum of the velocity of the cable relative to the shuttle and the velocity of the shuttle relative to the space station must be equal to the velocity of the cable relative to the space station.

Identify the Goal

The velocity of the cable relative to the space station.

Variables and Constants

<table>
<thead>
<tr>
<th>Known</th>
<th>Implied</th>
<th>Unknown</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \vec{v}_{cl} = 12 )</td>
<td>( \theta )</td>
<td>( v_{clx} )</td>
</tr>
<tr>
<td>( \vec{v}_{cs} = 3.0 \text{ m} / \text{s} )</td>
<td>( v_{csx} )</td>
<td>( v_{cly} )</td>
</tr>
<tr>
<td>[25(^\circ) to shuttle]</td>
<td>( v_{cly} )</td>
<td>( \vec{v}_{cl} )</td>
</tr>
</tbody>
</table>
Strategy and Calculations

Draw the vectors on a x-y-coordinate system (+y coincides with north).

Scale 1.0 cm : 3 m/s

Find the x- and y-components of the vectors \( \vec{v}_{cs} \) and \( \vec{v}_{sI} \).

\[
\begin{align*}
\vec{v}_{cs} &= 12 \text{ m/s [E]} \\
\vec{v}_{sI} &= 3.0 \text{ m/s [E25°S]} \\
\vec{v}_{cs} &= \vec{v}_{sI} + \vec{v}_{cl} \\
\vec{v}_{cs} &= 3.0 \text{ m/s [E25°S]} \\
\theta_{cs} &= 25° \\
\vec{v}_{cs} &= |\vec{v}_{cs}| \cos \theta_{cs} \\
\vec{v}_{cs} &= 3.0 \text{ m/s} \cos 25° \\
\vec{v}_{cs} &= 3.0 \text{ m/s} (0.9063) \\
\vec{v}_{cs} &= 2.72 \text{ m/s} \\
\vec{v}_{csy} &= |\vec{v}_{cs}| \sin \theta_{cs} \\
\vec{v}_{csy} &= 3.0 \text{ m/s} \sin 25° \\
\vec{v}_{csy} &= 3.0 \text{ m/s} \sin(0.4226) \\
\vec{v}_{csy} &= 1.27 \text{ m/s} \\
\end{align*}
\]

Add the components of \( \vec{v}_{cs} \) and \( \vec{v}_{sI} \) to obtain the components of \( \vec{v}_{cl} \).

<table>
<thead>
<tr>
<th>Vector</th>
<th>x-component (m/s)</th>
<th>y-component (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \vec{v}_{cs} )</td>
<td>2.72</td>
<td>-1.27</td>
</tr>
<tr>
<td>( \vec{v}_{ci} )</td>
<td>14.72</td>
<td>-1.27</td>
</tr>
</tbody>
</table>

Use the Pythagorean theorem to find the magnitude of \( \vec{v}_{cl} \), and find the angle the resultant makes with the x-axis.

\[
\begin{align*}
|\vec{v}_{cl}|^2 &= (\vec{v}_{clx})^2 + (\vec{v}_{cly})^2 \\
|\vec{v}_{clx}|^2 &= (14.72 \text{ m/s})^2 + (-1.27 \text{ m/s})^2 \\
|\vec{v}_{cl}^2| &= 216.7 \text{ m}^2/\text{s}^2 + 1.61 \text{ m}^2/\text{s}^2 \\
|\vec{v}_{clx}|^2 &= 218.3 \text{ m}^2/\text{s}^2 \\
|\vec{v}_{cly}| &= 14.78 \text{ m/s} \\
|\vec{v}_{cl}| &= 15 \text{ m/s} \\
\tan \theta &= -\frac{1.27}{14.72} \\
\theta &= 0.0863 \\
\theta &= \tan^{-1} 0.0863 \\
\theta &= 4.9°
\end{align*}
\]
Since the $x$-component is positive and the $y$-component is negative the vector and the angle lie in the fourth quadrant.
The velocity of the cable relative to the space station is $15 \text{ m/s}$ in a direction $4.9^\circ$ to the shuttle.

**Validate**
The velocity of the cable relative to the space station is correct since it should be greater than the velocity of the shuttle relative to the space station alone. The cable possesses the shuttle’s velocity as well as its own.

## Practice Problem Solutions
Student Textbook page 467

### 7. (a) Frame the Problem
- Make a diagram of the problem; use a coordinate system with East to the right and North at the top.
- Because the vector sum of the three forces is zero, the forces in each dimension must add to zero.
- Find the components of the third force that will make the sum of the $x$ and $y$-components of all the forces equal to zero.

### Identify the Goal
The force, $\vec{F}_3$, which will make the net force equal to zero.

### Variables and Constants

<table>
<thead>
<tr>
<th>Known</th>
<th>Unknown</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\vec{F}_1 = 154 \text{ N}[E22^\circ S]$</td>
<td>$\vec{F}_3$</td>
</tr>
<tr>
<td>$\vec{F}_2 = 203 \text{ N}[W22^\circ N]$</td>
<td></td>
</tr>
</tbody>
</table>

### Develop a Strategy
Draw each of the known force vectors with its tail on the origin of the coordinate system and determine the angle each vector makes with the nearest $x$ axis.

Find the $x$-component of each of the force vectors.

| $F_{1x} = |\vec{F}_1| \cos 22.0^\circ$ | $F_{2x} = -|\vec{F}_2| \cos 74.0^\circ$ |
| $F_{1x} = (154 \text{ N})(0.92718)$ | $F_{2x} = -(203 \text{ N})(0.27564)$ |
| $F_{1x} = 142.786 \text{ N}$ | $F_{2x} = -55.954 \text{ N}$ |

The angle is in the 4th quadrant so the $x$-component is positive.

The angle is in the 2nd quadrant so the $x$-component is negative.
Find the $y$-component of each of the force vectors.

$$F_{1y} = -|\vec{F}_1| \sin 22.0^\circ$$
$$F_{2y} = |\vec{F}_2| \sin 74.0^\circ$$
$$F_{1y} = -(154 \text{ N})(0.37461)$$
$$F_{2y} = (203 \text{ N})(0.96126)$$

The angle is in the 4th quadrant so the $y$-component is negative.

The angle is in the 2nd quadrant so the $y$-component is positive.

Sum the $x$ and $y$-components individually to find the components of the unknown vector.

$$F_{1x} + F_{2x} + F_{3x} = 0.0$$
$$F_{3x} = 0.0 - F_{1x} - F_{2x}$$
$$F_{3x} = 0.0 - (142.786 \text{ N}) - (-55.954 \text{ N})$$

$$F_{3x} = -86.832 \text{ N}$$

$$F_{1y} + F_{2y} + F_{3y} = 0.0$$
$$F_{3y} = 0.0 - F_{1y} - F_{2y}$$
$$F_{3y} = 0.0 - (-57.689 \text{ N}) - (195.136 \text{ N})$$

$$F_{3y} = -137.447 \text{ N}$$

Use the Pythagorean Theorem to find the magnitude of $\vec{F}_3$.

$$|\vec{F}_3| = (F_{3x})^2 + (F_{3y})^2$$

$$|\vec{F}_3| = (-86.832 \text{ N})^2 + (-137.447 \text{ N})^2$$

$$|\vec{F}_3| = 26431.474 \text{ N}^2$$

$$|\vec{F}_3| = 162.578 \text{ N}$$

$$|\vec{F}_3| \approx 163 \text{ N}$$

Use the tangent function and the magnitudes of the $x$- and $y$-components to find the angle $\theta_3$.

$$\tan \theta = \frac{F_{3y}}{F_{3x}}$$

$$\tan \theta = \frac{-137.447 \text{ N}}{-86.832 \text{ N}}$$

$$\tan \theta = 1.5829$$

$$\theta = \tan^{-1}(1.5829)$$

$$\theta = 57.72^\circ$$

$$\theta \approx 58^\circ$$

The third vector is $163 \text{ N}[W58^\circ S]$.

**Validate the Solution**

From a diagram, it is evident that the third vector will be in either the first or third quadrant, so the answer is reasonable.

7. (b) **Frame the Problem**

- Same as above.

**Identify the Goal**

The force, $\vec{F}_3$, which will make the net force equal to zero.
Variables and Constants

Identify the Variables

<table>
<thead>
<tr>
<th>Known</th>
<th>Unknown</th>
</tr>
</thead>
<tbody>
<tr>
<td>( F_1 = 782 \text{ N}[E12^\circ \text{N}] )</td>
<td>( F_3 )</td>
</tr>
<tr>
<td>( F_2 = 629 \text{ N}[W24^\circ \text{S}] )</td>
<td></td>
</tr>
</tbody>
</table>

Develop a Strategy

Draw each of the known force vectors with its tail on the origin of the coordinate system and determine the angle each vector makes with the nearest \( x \) axis.

Find the \( x \)-component of each of the force vectors.

\[
F_{1x} = |F_1| \cos 12.0^\circ \\
F_{1x} = (782 \text{ N})(0.97815) \\
F_{1x} = 764.91 \text{ N}
\]

The angle is in the 1st quadrant so the \( x \)-component is positive.

Find the \( y \)-component of each of the force vectors.

\[
F_{1y} = |F_1| \sin 12.0^\circ \\
F_{1y} = (782 \text{ N})(0.20791) \\
F_{1y} = 162.59 \text{ N}
\]

The angle is in the 1st quadrant so the \( y \)-component is positive.

Sum the \( x \) and \( y \)-components individually to find the components of the unknown vector.

\[
F_{1x} + F_{2x} + F_{3x} = 0.0 \\
F_{3x} = 0.0 - F_{1x} - F_{2x} \\
F_{3x} = 0.0 - (764.91 \text{ N}) - (-574.62 \text{ N}) \\
F_{3x} = -190.29 \text{ N}
\]

\[
F_{1y} + F_{2y} + F_{3y} = 0.0 \\
F_{3y} = 0.0 - F_{1y} - F_{2y} \\
F_{3y} = 0.0 - (162.59 \text{ N}) - (-255.84 \text{ N}) \\
F_{3y} = 93.25 \text{ N}
\]

Use the Pythagorean Theorem to find the magnitude of \( F_3 \).

\[
|F_3|^2 = (F_{3x})^2 + (F_{3y})^2 \\
|F_3|^2 = (-190.29 \text{ N})^2 + (93.25 \text{ N})^2 \\
|F_3|^2 = 44905.8 \text{ N}^2 \\
|F_3| = 211.91 \text{ N} \\
|F_3| \approx 212 \text{ N}
\]
Use the tangent function and the magnitudes of the $x$- and $y$-components to find the angle $\theta_3$.

\[
\tan \theta = \frac{F_{3y}}{F_{3x}}
\]

\[
\tan \theta = \frac{-137.447 \text{ N}}{-86.832 \text{ N}} = 1.5829
\]

\[
\theta = \tan^{-1}(1.5829) 
\]

\[
\theta = 57.72^\circ
\]

\[
\theta \equiv 58^\circ
\]

The third vector is 212 N[W26°N].

**Validate the Solution**

From a diagram, it is evident that the third vector will be in either the second or fourth quadrant, so the answer is reasonable.

7. (c) **Frame the Problem**

- Same as above.

**Identify the Goal**

The force, $\vec{F}_4$, which will make the net force equal to zero.

**Variables and Constants**

**Identify the Variables**

<table>
<thead>
<tr>
<th>Known</th>
<th>Unknown</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\vec{F}_1 = 48 \text{ N}[W81^\circ N]$</td>
<td>$\vec{F}_4$</td>
</tr>
<tr>
<td>$\vec{F}_2 = 61 \text{ N}[E63^\circ N]$</td>
<td></td>
</tr>
<tr>
<td>$\vec{F}_3 = 78 \text{ N}[E15^\circ S]$</td>
<td></td>
</tr>
</tbody>
</table>

**Develop a Strategy**

Draw each of the known force vectors with its tail on the origin of the coordinate system and determine the angle each vector makes with the nearest $x$ axis.

Find the $x$-component of each of the force vectors.

\[
F_{1x} = -|\vec{F}_1|\cos81.0^\circ
\]

\[
F_{1x} = -(48 \text{ N})(0.15643) = -7.5088 \text{ N}
\]

The angle is in the 2nd quadrant so the $x$-component is negative.

\[
F_{2x} = |\vec{F}_2|\cos63.0^\circ
\]

\[
F_{2x} = (61 \text{ N})(0.4540) = 27.693 \text{ N}
\]

The angle is in the 1st quadrant so the $x$-component is positive.

\[
F_{3x} = |\vec{F}_3|\cos15.0^\circ
\]

\[
F_{3x} = (78 \text{ N})(0.96592) = 75.342 \text{ N}
\]

The angle is in the 4th quadrant so the $x$-component is positive.
Find the $y$-component of each of the force vectors.

\[
F_{1y} = F_1 \sin 81.0^\circ \\
F_{2y} = F_2 \sin 63.0^\circ \\
F_{3y} = -F_3 \sin 15.0^\circ \\
F_{4y} = (48 \text{ N})(0.9877) \\
F_{4y} = (61 \text{ N})(0.8910) \\
F_{4y} = 47.409 \text{ N} \\
The angle is in the 2nd quadrant so the $y$-component is positive.
\]

Sum the $x$ and $y$-components individually to find the components of the unknown vector.

\[
F_{1x} + F_{2x} + F_{3x} + F_{4x} = 0.0 \\
F_{4x} = 0.0 - F_{1x} - F_{2x} - F_{3x} \\
F_{4x} = 0.0 - (-7.5088 \text{ N}) - (27.6934 \text{ N}) - (75.3422 \text{ N}) \\
F_{4x} = -95.527 \text{ N} \\
F_{1y} + F_{2y} + F_{3y} + F_{4y} = 0.0 \\
F_{4y} = 0.0 - F_{1y} - F_{2y} - F_{3y} \\
F_{4y} = 0.0 - (47.4090 \text{ N}) - (54.3514 \text{ N}) - (-20.1879 \text{ N}) \\
F_{4y} = -81.5725 \text{ N}
\]

Use the Pythagorean Theorem to find the magnitude of $\vec{F}_4$.

\[
|\vec{F}_4|^2 = (F_{4x})^2 + (F_{4y})^2 \\
|\vec{F}_4|^2 = (-95.527 \text{ N})^2 + (81.5725 \text{ N})^2 \\
|\vec{F}_4|^2 = 15779.4 \text{ N}^2 \\
|\vec{F}_4| = 125.62 \text{ N} \\
|\vec{F}_4| \approx 126 \text{ N}
\]

Use the tangent function and the magnitudes of the $x$- and $y$-components to find the angle $\theta_4$.

\[
\tan \theta = \frac{F_{4y}}{F_{4x}} \\
\tan \theta = \frac{-81.5725 \text{ N}}{-95.5268 \text{ N}} \\
\tan \theta = 0.8539 \\
\theta = \tan^{-1}(0.8539) \\
\theta = 40.49^\circ \\
\theta \approx 40^\circ
\]

The fourth vector is 126 N[\text{W}40^\circ\text{S}].

**Validate the Solution**

From a diagram, it is evident that the fourth vector will be in the third quadrant to balance the other force vectors, so the answer is reasonable.
8. Frame the Problem
- Caitlin acts as the equilibrating force, so the question is similar to the above.

Identify the Goal
The force, \( \vec{F}_C \), which will make the net force equal to zero.

Variables and Constants
Identify the Variables

<table>
<thead>
<tr>
<th>Known</th>
<th>Unknown</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \vec{F}_A = 15 , \text{N}[N58^\circ\text{E}] )</td>
<td>( \vec{F}_C )</td>
</tr>
<tr>
<td>( \vec{F}_B = 18 , \text{N}[S23^\circ\text{E}] )</td>
<td></td>
</tr>
</tbody>
</table>

Develop a Strategy
Draw each of the known force vectors with its tail on the origin of the coordinate system and determine the angle each vector makes with the nearest x-axis.

Find the x-component of each of the force vectors.

\[
F_{Ax} = |\vec{F}_A|\cos32^\circ \\
F_{Ax} = (15 \, \text{N})(0.8480) \\
F_{Ax} = 12.7207 \, \text{N}
\]

The angle is in the 1st quadrant so the x-component is positive.

Find the y-component of each of the force vectors.

\[
F_{Ay} = |\vec{F}_A|\sin32^\circ \\
F_{Ay} = (15 \, \text{N})(0.5299) \\
F_{Ay} = 7.9488 \, \text{N}
\]

The angle is in the 1st quadrant so the y-component is positive.

Calculations
Amy's force is 32\(^\circ\) to the positive x-axis.
Buffy's force is 67\(^\circ\) to the positive x-axis.

From the components, the equilibrating vector is in the second quadrant.

Sum the x and y-components individually to find the components of the unknown vector.

\[
\begin{align*}
F_{Ax} + F_{Bx} + F_{Cx} &= 0.0 \\
F_{Cx} &= 0.0 - F_{Bx} - F_{Bx} \\
F_{Cx} &= 0.0 - (12.7207 \, \text{N}) - (7.0332 \, \text{N}) \\
F_{Cx} &= -19.7539 \, \text{N} \\
F_{Ay} + F_{By} + F_{Cy} &= 0.0 \\
F_{Cy} &= 0.0 - F_{Ay} - F_{Ay} \\
F_{Cy} &= 0.0 - (7.9488 \, \text{N}) - (-16.5691 \, \text{N}) \\
F_{Cy} &= 8.6203 \, \text{N}
\end{align*}
\]
Use the Pythagorean Theorem to find the magnitude of $\vec{F}_C$.

$$|\vec{F}_C|^2 = (F_{Cx})^2 + (F_{Cy})^2$$

$$|\vec{F}_C|^2 = (-19.7539 \text{ N})^2 + (8.6203 \text{ N})^2$$

$$|\vec{F}_C|^2 = 464.526 \text{ N}^2$$

$$|\vec{F}_C| = 21.553 \text{ N}$$

$$|\vec{F}_C| \equiv 22 \text{ N}$$

Use the tangent function and the magnitudes of the $x$- and $y$-components to find the angle $\theta_3$.

$$\tan \theta = \frac{F_{Cy}}{F_{Cx}}$$

$$\tan \theta = \frac{8.6203 \text{ N}}{-19.7539 \text{ N}}$$

$$\tan \theta = -0.4364$$

$$\theta = \tan^{-1}(-0.4364)$$

$$\theta = -23.5757^\circ$$

$$\theta \equiv -24^\circ$$

Caitlin exerts a force of 22 N[W24°N].

**Validate the Solution**

From a diagram, it is evident that the third vector will be in either the second or third quadrant, so the answer is reasonable.

9. **Frame the Problem**

- The force of gravity on the traffic light is balanced by the tension force in the cables.
- Draw a free body diagram representing the forces on the traffic light.
- The traffic light is in equilibrium and has no net acceleration in either the vertical or horizontal directions.

**Identify the Goal**

The force that the cables, $\vec{F}_{\text{Cable}}$, exert on the traffic light to prevent it from falling.

**Variables and Constants**

**Identify the Variables**

<table>
<thead>
<tr>
<th>Known</th>
<th>Implied</th>
<th>Unknown</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m = 65 \text{ kg}$</td>
<td>$\vec{F}_{\text{Cable}}$</td>
<td></td>
</tr>
<tr>
<td>$\theta = 12^\circ$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\bar{g} = 9.81 \text{ m} / \text{s}^2$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Develop a Strategy**

Apply Newton’s second law in the $y$-direction.

Note from the hint that the tension in the two cables is equal.

**Calculations**

$$F_{\text{Cable1y}} + F_{\text{Cable2y}} - mg = 0$$

$$F_{\text{Cable1}} \sin 12^\circ + F_{\text{Cable2}} \sin 12^\circ = mg$$

$$2F_{\text{Cable}} \sin 12^\circ = mg$$

$$F_{\text{Cable}} = \frac{mg}{2 \sin 12^\circ}$$

$$F_{\text{Cable}} = \frac{(65 \text{ kg})(9.81 \text{ m} / \text{s}^2)}{2(0.2079)}$$

$$F_{\text{Cable}} \approx 1533.46$$

$$F_{\text{Cable}} \approx 1.5 \times 10^3 \text{ N}$$
The tension in the cables is $1.5 \times 10^3$ N. (Note that applying Newton's second law in the horizontal direction was unnecessary.

**Validate the Solution**

The tension in the cables is much greater than the weight of the light ($mg = 65 \text{ kg} \times 9.81 \text{ m/s}^2 = 638 \text{ N}$), as expected, to keep the light steady and resist additional forces due to wind, etc. Therefore, the answer is reasonable.

**Practice Problem Solutions**

Student Textbook pages 474–475

10. **Frame the Problem**

- Sketch a free body diagram of the box on the ramp.
- If the component of the force of gravity that is parallel to the ramp is greater than the maximum possible friction force, the box will slide down the ramp. Otherwise it will remain motionless.

**Identify the Goal**

(a) Whether the box will slide down the ramp or remain motionless.

(b) The applied force, $F_a$, needed to start the box moving, if it was motionless.

**Variables and Constants**

**Identify the Variables**

<table>
<thead>
<tr>
<th>Known</th>
<th>Implied</th>
<th>Unknown</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m = 45 \text{ kg}$</td>
<td>$g = 9.81 \text{ m/s}^2$</td>
<td>$F_a$</td>
</tr>
<tr>
<td>$\theta = 21^\circ$</td>
<td></td>
<td>$F_t$</td>
</tr>
<tr>
<td>$\mu_s = 0.42$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Develop a Strategy**

Find the $x$ component of the force of gravity.

**Calculations**

$\sin \theta = \frac{F_{gx}}{F_b}$

$F_{gx} = |F_b| \sin \theta$

$F_{gx} = mg \sin \theta$

$F_{gx} = (45 \text{ kg})(9.81 \text{ m/s}^2) \sin 21^\circ$

$F_{gx} = 158.20 \text{ N}$
Find the maximum possible force of static friction.

\[ F_{f(max)} = \mu_s F_N \]
\[ F_{f(max)} = \mu_s mg \cos \theta \]
\[ F_{f(max)} = (0.42)(45 \text{ kg})(9.81 \text{ m} \text{ s}^{-2}) \cos 21^\circ \]
\[ F_{f(max)} = 173.09 \text{ N} \]

(a) Since the force of friction is greater than the \(x\)-component of the force of gravity, the box will remain motionless.

Take the difference of the forces to determine the required force down the ramp to get the box to start to slide.

An applied force of 15 N is required to get the box to start to slide.

Validate the Solution
The ramp has only a shallow tilt, so it is reasonable that the box does not slide. It is also reasonable that a small nudge of 15 N will get it started.

11. Frame the Problem
- Sketch a free body diagram of the container on the ramp. (It’s the same as in the previous question.)
- When the component of the force of gravity that is parallel to the ramp becomes greater than the maximum possible friction force, the container will begin to slide down the ramp.
- When the container begins sliding, Newton’s second law can be applied to determine the acceleration down the ramp.

Identify the Goal
(a) The angle, \(\theta\), of the ramp at which the container just starts to slide.
(b) The acceleration, \(a_x\), of the container just after it started to slide.

Variables and Constants

Identify the Variables

<table>
<thead>
<tr>
<th>Known</th>
<th>Implied</th>
<th>Unknown</th>
</tr>
</thead>
<tbody>
<tr>
<td>(m = 61 \text{ kg})</td>
<td>(g = 9.81 \text{ m} \text{ s}^{-2})</td>
<td>(\theta)</td>
</tr>
<tr>
<td>(\mu_s = 0.37)</td>
<td></td>
<td>(a_x)</td>
</tr>
<tr>
<td>(\mu_k = 0.18)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Develop a Strategy
Find the \(x\) component of the force of gravity.

Calculations
\[ \sin \theta = \frac{F_{gx}}{|F_x|} \]
\[ F_{gx} = |F_x| \sin \theta \]
\[ F_{gx} = mg \sin \theta \]
\[ F_{f(max)} = \mu_s F_N \]
\[ F_{f(max)} = \mu_s mg \cos \theta \]
The container will begin to slide when \( F_{gx} > F_{f(max)} \). So, to find the angle, set the ratio of these to 1 and solve for the angle.

\[
\frac{F_{gx}}{F_{f(max)}} = 1 = \frac{mg \sin \theta}{\mu_s mg \cos \theta}
\]

\[
\tan \theta = \frac{\mu_s}{\mu_k}
\]

\[
\tan \theta = \mu_k = 0.37
\]

\[
\theta = \tan^{-1}(0.37)
\]

\[
\theta = 20.3^\circ
\]

(a) The container will begin to slide when the ramp has an angle of 20.3°.

As soon as the container begins to slide, the coefficient of kinetic friction replaces the coefficient of static friction. Find the acceleration in the \( x \) direction (down the ramp) by applying Newton’s second law to the components of force in the \( x \) direction.

The component of the gravitational force and the frictional force act in opposite directions so they have opposite signs.

(b) After the container begins to slide, its acceleration would be 1.7 m/s² down the ramp.

Validate the Solution

The angle for the ramp is reasonable (and similar to those given in previous questions). The value obtained for the acceleration is much less than the acceleration due to gravity (or in free fall), so it is reasonable.

12. Frame the Problem

- Sketch a free body diagram of the container on the ramp.
- By applying Newton’s second law to the components of forces in the \( x \) and \( y \) directions, the unknown forces can be determined from the known forces.
- Pushing the container at constant velocity implies that the acceleration is zero.

Identify the Goal

The applied force, \( F_a \), required the push the container up the ramp at a constant velocity.

Variables and Constants

<table>
<thead>
<tr>
<th>Known</th>
<th>Implied</th>
<th>Unknown</th>
</tr>
</thead>
<tbody>
<tr>
<td>( m = 55 \text{ kg} )</td>
<td>( g = 9.81 \text{ m/s}^2 )</td>
<td>( F_a )</td>
</tr>
<tr>
<td>( \mu_k = 0.18 )</td>
<td>( a_x = 0 )</td>
<td></td>
</tr>
<tr>
<td>( \theta = 33^\circ )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Develop a Strategy
Apply Newton's second law in the $y$-direction.
Apply Newton's second law in the $x$-direction, noting that the acceleration in this direction is zero.
Substitute the normal force and expand the terms.
Substitute numerical values and solve.

Calculations

\[ F_N = mg \cos \theta \]

\[ -F_x - F_t + F_a = ma_x = 0 \]

\[ F_a = F_t + F_y \]

\[ F_a = \mu_k F_N + mg \sin \theta \]

\[ F_a = \mu_k mg \cos \theta + mg \sin \theta \]

\[ F_a = mg (\mu_k \cos \theta + \sin \theta) \]

\[ F_a = (55 \text{ kg})(9.81 \text{ m/s}^2)(0.23 \cos 33^\circ + \sin 33^\circ) \]

\[ F_a = 397.94 \text{ N} \]

\[ F_a \approx 4.0 \times 10^2 \text{ N} \]

An applied force of $4.0 \times 10^2 \text{ N}$ is required.

Validate the Solution
The force required is slightly less than the weight of the container ($mg = 55 \text{ kg} \times 9.81 \text{ m/s}^2 = 540 \text{ N}$). Because the ramp is relatively steep, a significant force is required to move the container along at a constant velocity, so the answer is reasonable.

For comparison, moving the container at constant velocity along the ground would require a force, $F_a = \mu mg = 0.23 \times 55 \text{ kg} \times 9.81 \text{ m/s}^2 = 124 \text{ N}$, so, clearly, the steepness of the ramp makes it necessary to apply a much larger force.

13. Frame the Problem
- Sketch a free body diagram of the crate on the ramp.
- By applying Newton's second law to the components of the forces in the $x$ and $y$-directions, the unknown forces can be determined from the known forces.
- In order to start the crate moving, the coefficient of static friction is required.

Identify the Goal
(a) The applied force, $F_a$, required the start the crate moving up the ramp, when pushing parallel to the ground.
(b) The applied force, $F_a'$, required the start the crate moving up the ramp, when pushing parallel to the ramp.
Variables and Constants
Identify the Variables

Known Implied Unknown
\[ m = 85 \text{ kg} \quad \bar{g} = 9.81 \text{ m/s}^2 \quad F_a \]
\[ \mu_s = 0.46 \]
\[ \theta = 28^\circ \]

Develop a Strategy

Apply Newton's second law in the \( y \)-direction. Note the hint, which says that a component of the applied force is perpendicular to the ramp and thus contributes to the normal force.

Apply Newton's second law in the \( x \)-direction, noting that the acceleration in this direction is zero.

Substitute the normal force and expand the terms.

Substitute numerical values and solve.

\[
F_a = \frac{\mu_s mg \cos \theta + mg \sin \theta}{\cos \theta - \mu_s \sin \theta}
\]

\[
F_a = \frac{(0.46)(85 \text{ kg})(9.81 \text{ m/s}^2) \cos 28^\circ + (85 \text{ kg})(9.81 \text{ m/s}^2) \sin 28^\circ}{\cos 28^\circ + (0.46) \sin 28^\circ}
\]

\[
F_a = 1094.68 \text{ N}
\]

\[
F_a \cong 1.1 \times 10^3 \text{ N}
\]

(a) An applied force of \( 1.1 \times 10^3 \text{ N} \) is required.

When the worker pushes parallel to the ramp, the applied force is parallel to the ramp. So, the terms in
the above equation (derived from the \( x \)-components) that contain \( F \cos \theta \) or \( F \sin \theta \) should be replaced by \( F_a \) and 0, respectively, because the pushing angle is now zero. Effectively, this sets the denominator in the above equation to 1.

Substitute numerical values and solve.

\[
F_a = \mu mg \cos \theta + mg \sin \theta
\]

\[
F_a = (0.46)(85 \text{ kg})(9.81 \text{ m/s}^2) \cos 28^\circ + (85 \text{ kg})(9.81 \text{ m/s}^2) \sin 28^\circ
\]

\[
F_a = 730.14 \text{ N}
\]

\[
F_a \approx 7.3 \times 10^2 \text{ N}
\]

(b) To start the crate moving up the ramp by pushing parallel to the ramp requires a force of \( 7.3 \times 10^2 \text{ N} \).

Validate the Solution
The units work out to be newtons in each case, as expected. By pushing at a more appropriate angle in the second case, less force is required, so the answer is reasonable.

Practice Problem Solutions
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14. Conceptualize the Problem
- Begin framing the problem by drawing a free-body diagram of the forces acting on the girl.
- The tension of the rope pulls her up, while the force of gravity pulls her down.
- Let “up” be positive and “down” be negative.
- Because the girl accelerates downward, it is expected that the sign of the acceleration will be negative.
- The acceleration can be found by applying Newton’s second law.

Identify the Goal
The acceleration, \( \vec{a} \), of the child as she is being lowered

Identify the Variables

<table>
<thead>
<tr>
<th>Known</th>
<th>Implied</th>
<th>Unknown</th>
</tr>
</thead>
<tbody>
<tr>
<td>( m = 32 \text{ kg} )</td>
<td>( \vec{g} = -9.81 \text{ m/s}^2 ) [up]</td>
<td>( \vec{a} )</td>
</tr>
<tr>
<td>( \vec{F}_t = 253 \text{ N}[\text{up}] )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Develop a Strategy

Apply Newton’s second law to the free-body diagram.

Substitute and solve.

The child has an acceleration of about \(-1.9\) m/s\(^2\).[up].

Validate the Solution

The child is being lowered from a rope held by her parent, so it is expected that her acceleration will be downward, and much less than the acceleration due to gravity, which it is.

15. Solution is similar to Practice Problem 14.

The climber has an acceleration of 2.5 m/s\(^2\)[down].

16. Conceptualize the Problem

- Begin framing the problem by drawing a free-body diagram of the forces acting on the mass on the scale.
- The forces acting on the mass are gravity (\(\vec{F}_g\)), which is downwards, and the normal force of the scale, which is upwards.
- According to Newton’s third law, when the mass exerts a force (\(F_{MS}\)) on the scale, it exerts an equal and opposite force (\(F_{SM}\)) on the mass. Therefore, the reading on the scale is the same as the force that the mass exerts on the scale. In the same way, the tension in the hoist rope will also be the same.
- The motion is in one direction so let “up” be positive and “down” be negative.
- Because the motion is in one dimension, perform calculations with magnitudes only.
- Also, because the mass is just starting to move, the sign of the acceleration will indicate the direction of the motion (the mass is accelerating, not decelerating).
- Apply Newton’s second law to find the acceleration of the mass.

Identify the Goals

(a) the direction of motion.
(b) the acceleration, \(a\), of the mass.
(c) the tension, \(F_T\), of the hoist rope.

Identify the Variables

<table>
<thead>
<tr>
<th>Known</th>
<th>Implied</th>
<th>Unknown</th>
</tr>
</thead>
<tbody>
<tr>
<td>(m = 10.0) kg</td>
<td>(g = 9.81) m/s(^2)</td>
<td>(F_{MS})</td>
</tr>
<tr>
<td>(\vec{F}_{SM} = 87) N</td>
<td>(\vec{F}_g)</td>
<td>(F_T)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(a)</td>
</tr>
</tbody>
</table>
Develop a Strategy
Apply Newton’s second law to the free-body diagram.

Substitute and solve.

(a) The acceleration is negative, so the motion is downwards.

(b) The acceleration of the mass is \(-1.1 \text{ m/s}^2\). 

(c) From Newton’s third law, the tension in the hoist rope will be the same as the force the mass exerts on the scale, or 87 N.

Validate the Solution
Because of the negative sign of the acceleration (i.e. directed downwards), the reading on the scale is slightly less than it would be if the hoist had zero acceleration. In that case, the scale would read, \((10.0 \text{ kg})(9.81 \text{ m/s}^2) = 98.1 \text{ N}\). This agrees with the experience of riding in an elevator: as it starts travelling downwards, you feel lighter.

17. Solution is similar to Practice Problem 16.
   The tension of the backpack strap is about \(1.7 \times 10^2 \text{ N}\).

18. Solution is similar to Practice Problem 16.
   The elevator has a maximum acceleration of about \(1.8 \text{ m/s}^2\).

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19. Conceptualize the Problem
- Begin framing the problem by drawing free-body diagrams. Draw one diagram of the system moving as a unit and diagrams of each of the two individual objects.
- Let the negative direction point from the centre to the 3.8 kg mass and the positive direction point from the centre to the 4.2 kg mass. (If the rope was stretched out horizontally, the 3.8 kg mass would be to the left and the 4.2 kg mass would be to the right.) That is, “down” is positive for the 4.2 kg mass. See Figure 10.12 in the Student Text, and the Sample Problem, Motion of Connected Objects.
- Both objects move with the same acceleration. The heavier mass will move downwards and the lighter mass will move upwards.
- The force of gravity acts on both objects.
- The tension is constant throughout the rope.
- The rope exerts a force of equal magnitude and opposite direction on each object.
- When you isolate the individual objects, the tension in the rope is one of the forces acting on the object.
- Newton’s second law applies to the combination of the two objects and to each individual object.
Problem Tip
Vectors should always be written with both a magnitude and a direction. In problems such as the Atwood machine or Fletcher’s trolley, the directions are usually defined from the centre of the rope and by considering the rope as stretched out. Thus, students may have two pictures in their minds: the masses hanging on either side of the pulley, and the rope stretched horizontally. Depending on which picture they have, they may use terms like “right” and “left”, or “up” and “down” to define the directions of the vectors, which can be confusing — for example, talking about gravity as acting to the right! Therefore, it is perhaps simplest to define the coordinate system initially, and then to work with the magnitudes of the vectors in the calculations.

Identify the Goal
The acceleration, \(\vec{a}\), of the two objects
The tension, \(\vec{F}_T\), in the rope

Identify the Variables

<table>
<thead>
<tr>
<th>Known</th>
<th>Implied</th>
<th>Unknown</th>
</tr>
</thead>
<tbody>
<tr>
<td>(m_1) = 3.8 kg</td>
<td>(g = 9.81) m/s(^2)</td>
<td>(\vec{F}_T)</td>
</tr>
<tr>
<td>(m_2) = 4.2 kg</td>
<td></td>
<td>(\vec{a})</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(\vec{F}_{g1})</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(\vec{F}_{g2})</td>
</tr>
</tbody>
</table>

Develop a Strategy
Apply Newton’s second law to the combination of masses to find the acceleration.
As the motion can be considered to be in one dimension (if the rope is stretched out), the calculations can be done with magnitudes.
Substitute and solve.

Calculations

\[
\begin{align*}
F &= ma \\
F_{g1} + F_{g2} &= (m_1 + m_2)a \\
-m_1g + m_2g &= (m_1 + m_2)a \\
a &= \frac{(m_2 - m_1)g}{m_1 + m_2}
\end{align*}
\]

\[
\begin{align*}
a &= \frac{(4.2\ \text{kg} - 3.8\ \text{kg})(9.81\ \text{m/s}^2)}{4.2\ \text{kg} + 3.8\ \text{kg}} \\
a &\approx 0.4905\ \text{m/s}^2 \\
a &\approx 0.49\ \text{m/s}^2
\end{align*}
\]

The acceleration of the masses is about 0.49 m/s\(^2\).
The positive sign indicates the system accelerates in a clockwise direction.
Apply Newton’s second law to \(m_1\) and solve for tension.
Substitute and solve.

The tension in the rope is about 39 N. The positive sign indicates the tension on \(m_1\) acts to the right.

Validate the Solution
You can test your solution by applying Newton’s second law to the second mass.
\[ F = ma \]
\[ F_{g2} + F_T = m_2a \]
\[ F_T = m_2a - m_2g \]
\[ F_T = m_2(a - g) \]
\[ F_T = (4.2 \text{ kg})(0.4905 \text{ m/s}^2 - 9.81 \text{ m/s}^2) \]
\[ F_T = -39.14 \text{ N} \]
\[ F_T \approx -39 \text{ N} \]

The magnitudes of the tensions calculated independently from each of the masses agree. Also, notice that the application of Newton's second law correctly gave the direction for the force on the second mass (to the left).

20. Solution is similar to Practice Problem 19.
The mass of the second object is about 14 kg.
The tension in the rope is about 75 N[to the right].

21. Solution is similar to Practice Problem 19.
The acceleration of the system is about 1.6 m/s²[clockwise].
The mass of the second object is about 62 kg.

22. Solution is similar to Practice Problem 19.
The applied force is about 17 N.

23. Solution is similar to Practice Problem 19.
Both gymnasts accelerate upwards at 1.0 m/s².

**Practice Problem Solutions**
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24. Conceptualize the Problem
- Make a simplified diagram of the connected masses and assign forces.
- Visualize the light string in a straight configuration.
- Sketch free-body diagrams of the forces acting on each object and of the forces acting on the combined objects.
- The trolley experiences a horizontal force due to the tension in the string and vertical forces of gravity and the normal force, which act opposite to one another. See Figure 10.13 in the Student Text, and the Sample Problem, Connected Objects.
- The suspended mass experiences the force of gravity and the normal force, which act opposite to one another.
- The force causing the acceleration of both masses is the force of gravity acting on the suspended mass.
- Newton's second law applies to the combined masses and to each individual mass.
- Let left be the negative direction and right be the positive direction.

**Identify the Goal**
(a) The tension, \( F_T \), in the string when the suspended mass is released
(b) The acceleration, \( \vec{a} \), of the trolley
Identify the Variables

<table>
<thead>
<tr>
<th>Known</th>
<th>Implied</th>
<th>Unknown</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_1 = 1.90 \text{ kg}$</td>
<td>$g = 9.81 \text{ m/s}^2$</td>
<td>$\vec{F}_N$, $\vec{F}_T$</td>
</tr>
<tr>
<td>$m_2 = 0.500 \text{ kg}$</td>
<td></td>
<td>$\vec{a}$, $\vec{F}<em>{g1}$, $\vec{F}</em>{g2}$</td>
</tr>
</tbody>
</table>

Develop a Strategy

Apply Newton’s second law to the combined masses to find the acceleration. As the motion can be considered to be in one dimension (if the string is stretched out), the calculations can be done with magnitudes (see also the Problem Tip in Practice Problem 19).

Substitute and solve.

(a) The tension in the string is $3.88 \text{ N}$. Because the sign is positive, the tension acting on $m_1$ is to the right.

(b) The acceleration is about $2.04 \text{ m/s}^2$, to the right.

Apply Newton’s second law to the trolley (mass 1) to find the tension. Use magnitudes in the calculation.

Validate the Solution

The acceleration of the combined masses is less than $9.81 \text{ m/s}^2$, which is reasonable since only part of the mass is subject to unbalanced gravitational forces. The tension can be verified by applying Newton’s second law to the suspended mass.

The negative sign indicates that the tension on the suspended mass acts upwards.

25. Solution is similar to Practice Problem 24.

The glider travels to the end of the track in $0.67 \text{ s}$.

26. Frame the Problem

- Sketch the apparatus in its correct configuration with forces added. Add a coordinate system. In this case, let the direction to the right be positive.
- Then sketch the apparatus with the string in a straight line.
- Make free body diagrams of the bodies individually.
- Only block 2 is experiencing friction with the table.
- Due to the different masses on either ends of the system, the tensions in strings are not necessarily the same.
- Despite the different tensions, the system will move with uniform acceleration.
- The masses on either ends of the system feel the force of gravity and the tensions in the strings.
- Apply Newton’s second law to each free body diagram individually in both the x- and y-directions. Examine the set of equations and determine how best to solve for the unknowns.
- Once the acceleration is found, use a kinematical equation to find the time it takes the mass to fall.

**Identify the Goal**
The time interval, $\Delta t$, it takes for the mass, $m_3$, to reach the ground once the system starts moving.

**Variables and Constants**

**Identify the Variables**

<table>
<thead>
<tr>
<th>Known</th>
<th>Implied</th>
<th>Unknown</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_1 = 228 \text{ g} = 0.228 \text{ kg}$</td>
<td>$g = 9.81 \text{ m/s}^2$</td>
<td>$a$</td>
</tr>
<tr>
<td>$m_2 = 615 \text{ g} = 0.615 \text{ kg}$</td>
<td></td>
<td>$\Delta t$</td>
</tr>
<tr>
<td>$m_3 = 455 \text{ g} = 0.455 \text{ kg}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\mu_k = 0.26$</td>
<td></td>
<td>$\Delta d = 1.95 \text{ m}$</td>
</tr>
</tbody>
</table>

**Develop a Strategy**

Apply Newton’s second law to each free body diagram, in both the x- and y-directions.

For $m_1$.
(Only the x-direction is required.)

For $m_2$.
Note the acceleration in the y-direction is zero.

For $m_3$.
(Only the x-direction is required.)

**Calculations**

For $m_1$,

$F_{T1} - m_1g = m_1a$  \hspace{1cm} (1)

For $m_2$.

$x$: $F_{T2} - F_{T1} - F_i = m_2a$

or, $F_{T2} - F_{T1} - \mu_k F_N = m_2a$  \hspace{1cm} (2a)

$y$: $F_N - m_2g = 0$  \hspace{1cm} (2b)

For $m_3$.

$m_3g - F_{T2} = m_3a$  \hspace{1cm} (3)
By inspection, equations (1), (2b) and (3) can be rearranged and substituted into equation (2a). Collect like terms on each side of the equation. Solve for the acceleration. Substitute numerical values and solve.

The system accelerates at 0.507 m/s².

The units work out properly in both cases. The time interval of almost 3 s to fall about 2 m is very reasonable.

27. Frame the Problem
- Sketch the apparatus in its correct configuration with forces added. Add a coordinate system. In this case, let the direction to the right be positive.
- Then sketch the apparatus with the string in a straight line.
- Make free body diagrams of the bodies individually and as one system.
- Block 1 is experiencing friction with the ramp.
- Both a component of the gravitational force and a frictional force on the block are creating a force in the negative direction.
- For \( m_2 \), the tension operates in the negative direction and the gravitational force operates in the positive direction.
- Find the acceleration and use the definition of acceleration to find the velocity (or speed).

Identify the Goal
(a) The speed, \( v \), of the masses 2.5 s after they start moving.
(b) The tension in the string, \( F_T \), while they are moving.
Variables and Constants

Identify the Variables

<table>
<thead>
<tr>
<th>Known</th>
<th>Implied</th>
<th>Unknown</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_1 = 145 \text{ g} = 0.145 \text{ kg}$</td>
<td>$\ddot{g} = 9.81 \text{ m/s}^2$</td>
<td>$\ddot{a}$</td>
</tr>
<tr>
<td>$m_2 = 85 \text{ g} = 0.085 \text{ kg}$</td>
<td>$v_i = 0.0 \text{ m/s}$</td>
<td>$\nu$</td>
</tr>
<tr>
<td>$\mu_k = 0.18$</td>
<td></td>
<td>$\vec{F}_T$</td>
</tr>
<tr>
<td>$\Delta t = 2.5 \text{ s}$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Develop a Strategy

To find the frictional force, you need the normal force. Apply Newton’s second law in the direction perpendicular to the plane.

Since the two masses have the same acceleration, define the system as the combination of the masses.

Find the acceleration from Newton’s second law in the direction of the string when it is drawn horizontal. Call this the $x$ direction. Substitute in the normal force from the above, and expand terms.

Solve for the acceleration.

Substitute numerical values and solve.

Calculations

$F_x = ma_x$

$F_{g2x} - F_{g1x} - F_i = (m_1 + m_2)a_x$

$m_2g - m_1g \sin \theta - \mu_k F_N = (m_1 + m_2)a_x$

$a_x = \frac{m_2g - m_1g \sin \theta - \mu_k(m_1g \sin \theta)}{(m_1 + m_2)}$

$a_x = \frac{m_2 - m_1 \sin \theta - \mu_k m_1 \sin \theta}{(m_1 + m_2)}$

$a_x = \frac{0.085 \text{ kg} - (0.145) \sin 22^\circ - (0.18)(0.145 \text{ kg}) \cos 22^\circ}{(0.145 \text{ kg} + 0.085 \text{ kg})} \times 9.81 \text{ m/s}^2$

$a_x = 0.2765 \text{ m/s}^2$
Use the definition of acceleration to find the velocity.

The initial velocity is zero.

\[ a_s = \frac{\Delta v}{\Delta t} \]

\[ \Delta v = (v - v_i) = a_s \Delta t \]

\[ v = (0.2765 \text{ m} / \text{s}^2)(2.5 \text{ s}) \]

\[ v = 0.6912 \text{ m} / \text{s} \]

\[ v \equiv 0.69 \text{ m} / \text{s} \]

(a) The speed of the masses after they start to move will be 0.69 m/s.

By examining the free body diagram for the mass \( m_2 \), the tension can be found.

Rearrange and solve for the tension.

Substitute numerical values and solve.

(b) The tension in the string while they are moving is 0.81 N.

Validate the Solution

The units work out properly in both cases. The speed is equivalent to 69 cm/s which seems reasonable considering that the ramp is not steep so the free hanging mass pulls it along quickly.

28. Frame the Problem
- Sketch the apparatus in its correct configuration with forces added. Add a coordinate system. In this case, let the direction to the right be positive.
- Then sketch the apparatus with the string in a straight line.
- Make free body diagrams of the bodies individually and as one system.
- Block 1 is experiencing friction with the ramp.
- Both a component of the gravitational force and a frictional force on the block are creating a force in the negative direction.
- For \( m_2 \), the tension operates in the negative direction and the gravitational force operates in the positive direction.

Identify the Goal
(a) The force to apply, \( F_a \), to make the objects start to move.
(b) The acceleration, \( a \), after the objects start to move.
(c) The tension, \( F_T \), in the string when the objects are moving.

Variables and Constants

Identify the Variables

<table>
<thead>
<tr>
<th>Known</th>
<th>Implied</th>
<th>Unknown</th>
</tr>
</thead>
<tbody>
<tr>
<td>( m_1 = 725 \text{ g} = 0.725 \text{ kg} )</td>
<td>( \bar{g} = 9.81 \text{ m} / \text{s}^2 )</td>
<td>( F_a )</td>
</tr>
<tr>
<td>( m_2 = 595 \text{ g} = 0.595 \text{ kg} )</td>
<td>( \mu_k = 0.12 )</td>
<td>( \bar{a} )</td>
</tr>
<tr>
<td>( \mu_s = 0.47 )</td>
<td></td>
<td>( F_T )</td>
</tr>
</tbody>
</table>

\[ F_a = \sum F_x - m_1 g - \mu_k m_1 g - \mu_s m_1 g \]

\[ F_T = m_2 \bar{a} - \bar{g} m_2 - \mu_k m_2 \bar{g} \]

\[ F_T = 0.085 \text{ kg}(9.81 \text{ m} / \text{s}^2 - 0.2765 \text{ m} / \text{s}^2) \]

\[ F_T = 0.810 \text{ N} \]

\[ F_T \equiv 0.81 \text{ N} \]
**Develop a Strategy**

To find the frictional force, you need the normal force.

Apply Newton’s second law in the direction perpendicular to the plane.

Save this equation until it is needed.

Since the two masses have the same acceleration, define the system as the combination of the masses. Note that until the system begins to move, the acceleration is zero.

Find the applied force from Newton’s second law in the direction of the string when it is drawn horizontal. Call this the \( x \) direction. Substitute in the normal force from the above, and expand terms.

Solve for the applied force.

Substitute numerical values and solve.

(a) The applied force is 0.91 N.

To calculate the acceleration after the system starts to move, assume the applied force is now zero and proceed as above.
(b) The acceleration of the masses after they start to move will be 0.87 m/s². 

By examining the free body diagram for the mass \( m_2 \), the tension can be found.

Rearrange and solve for the tension.

Substitute numerical values and solve.

(c) The tension in the string while they are moving is 5.3 N.

Validate the Solution
The units work out properly in each case. The ramp is relatively steep, so a low value for the acceleration is expected. The acceleration is less than 1/10 of the acceleration due to gravity (free fall), which seems reasonable.

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29. Frame the Problem
- Sketch the situation, indicating the direction of the force.
- Use the equation for torque to determine the magnitude of the torque.

Identify the Goal
The torque, \( \tau \), exerted by your biceps muscle.

Variables and Constants
Identify the Variables

Known Unknown
\( r \perp = 0.052 \text{ m} \) \( \tau \)
\( F = 1250 \text{ N} \)

Develop a Strategy
The lever arm is the distance from the hinge of the elbow to where the biceps attaches.
Simply apply the equation for torque.
The torque is 65 N\( \cdot \)m.

Validate the Solution
The units are newton metres, the correct unit for torque.

30. Frame the Problem
- Sketch the situation, indicating the direction of the force.
- Use the equation for torque to determine the magnitude of the torque.

Identify the Goal
The torque, \( \tau \), exerted by the painter’s weight on the ladder.
Variables and Constants

Identify the Variables

<table>
<thead>
<tr>
<th>Known</th>
<th>Unknown</th>
</tr>
</thead>
<tbody>
<tr>
<td>( r_1 = 0.052 \text{ m} )</td>
<td>( \tau )</td>
</tr>
<tr>
<td>( F = 1250 \text{ N} )</td>
<td></td>
</tr>
</tbody>
</table>

Develop a Strategy

The lever arm is the distance from the hinge of the elbow to where the biceps attaches. Simply apply the equation for torque. The torque is 65 N·m.

Validate the Solution

The units are newton metres, the correct unit for torque.

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31. Frame the Problem

- Sketch the situation, indicating the directions of the forces.
- Let the axis of rotation be the fulcrum. This will simplify the calculations (compared to choosing the bolt as the axis of rotation).
- Use the equation for rotational equilibrium to find the force the bolt exerts on the board.

Identify the Goal

The force, \( F_{\text{bolt}} \), the bolt must exert on the diving board to hold it in place.

Variables and Constants

Identify the Variables

<table>
<thead>
<tr>
<th>Known</th>
<th>Implied</th>
<th>Unknown</th>
</tr>
</thead>
<tbody>
<tr>
<td>( m_{\text{diver}} = 54 \text{ kg} )</td>
<td>( g = 9.81 \text{ m/s}^2 )</td>
<td>( F_{\text{bolt}} )</td>
</tr>
<tr>
<td>( m_{\text{board}} = 25 \text{ kg} )</td>
<td>( l = 3.8 \text{ m} )</td>
<td></td>
</tr>
<tr>
<td>( l_{\text{fulcrum}} = 1.3 \text{ m} )</td>
<td>( \ell_1 = 2.5 \text{ m} )</td>
<td></td>
</tr>
</tbody>
</table>

Develop a Strategy

Begin by writing the equation for rotational equilibrium, using the fulcrum as the axis.
The lever arm for the diver is then
\[ l_1 = 3.8 \text{ m} - 1.3 \text{ m} = 2.5 \text{ m}. \]
The lever arm for the board, assuming its weight acts at its midpoint, is,
\[ l_2 = 3.8 \text{ m}/2 - 1.3 \text{ m} = 0.6 \text{ m}. \]
Because the lever arm for the force acting at the fulcrum is zero, this force does not come into the calculation.

Substitute numerical values and solve.
The bolt exerts a force of \( 1.1 \times 10^3 \text{ N} \) on the board.

Note that at this point the problem is solved and the equations of translational equilibrium do not need to be applied.

**Validate the Solution**
The units are newtons, as required. The bolt exerts a force greater than the weight of the diver and the board, as expected. The value seems reasonable.

### 32. Frame the Problem
- Sketch the situation, indicating the directions of the forces.
- Let the axis of rotation be the elbow joint.
- Use the equation for *rotational equilibrium* to find the force the biceps exerts on the forearm.

### Identify the Goal
The force, \( F_{\text{biceps}} \), the biceps muscle exerts on your forearm to hold it in place.

### Variables and Constants

#### Identify the Variables

<table>
<thead>
<tr>
<th>Known</th>
<th>Implied</th>
<th>Unknown</th>
</tr>
</thead>
<tbody>
<tr>
<td>( m_{\text{arm}} ) = 2.3 kg</td>
<td>( g = 9.81 \text{ m/s}^2 )</td>
<td>( F_{\text{biceps}} )</td>
</tr>
<tr>
<td>( m_{\text{ball}} ) = 16 kg</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( l_1 ) = 0.052 m</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( l_2 ) = 0.15 m</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( l_3 ) = 0.34 m</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Develop a Strategy
Begin by writing the equation for rotational equilibrium, using the elbow joint as the axis.
Rearrange and isolate the force exerted by the biceps.
Note that the lever arm for the force exerted by the humerus bone is zero, so this force drops out of the calculation.
Substitute numerical values and solve.
The force exerted by the biceps muscle on the forearm is $1.1 \times 10^3$ N.

Validate the Solution
The units are newtons, as required. The force is required to be large because its lever arm is small, so the answer is reasonable.

33. Frame the Problem
- Sketch the situation, indicating the directions of all the forces.
- Let the contact of the beam with the pole be the axis of rotation.
- Use the equation for rotational equilibrium to find the tension in the cable.
- Use the equations for translational equilibrium in the $x$ and $y$ directions to find the $x$- and $y$- components of the force of the pole on the beam.

Identify the Goal
(a) The tension, $\vec{F}_T$, in the cable.
(b) The force exerted on the beam by the pole, $\vec{F}_{pole}$. 
Develop a Strategy
Let the contact of the beam with the pole be the axis.
Begin by writing the equation for rotational equilibrium.
Rearrange and isolate the tension force.
Note that the length of the beam cancels.
Substitute numerical values and solve.

(a) The tension in the cable is \(4.6 \times 10^2\) N.

Apply the equations of translational equilibrium to find the force the pole exerts on the beam.
First for the \(x\)-components.

Then for the \(y\)-components.
Use the Pythagorean theorem to find the magnitude of the force exerted by the pole.

\[ \vec{F}_{\text{pole}}^2 = F_{\text{pole}}^2 + F_{\text{pole}}^{\text{ex}}^2 \]

\[ \vec{F}_{\text{pole}}^2 = (388.98 \text{ N})^2 + (17.166 \text{ N})^2 \]

\[ \vec{F}_{\text{pole}}^2 = 151600.1 \text{ N}^2 \]

\[ \vec{F}_{\text{pole}} = 389.36 \text{ N} \]

\[ \vec{F}_{\text{pole}} \approx 3.9 \times 10^2 \text{ N} \]

Use the tangent function to find the angle that the force of the pole makes with the beam.

\[ \tan \theta = \frac{F_{\text{pole}}^{\text{ex}}}{F_{\text{pole}}} \]

\[ \theta = \tan^{-1} \frac{17.166 \text{ N}}{388.98 \text{ N}} \]

\[ \theta = \tan^{-1} 0.04413 \]

\[ \theta = 2.53^\circ \]

\[ \theta \approx 2.5^\circ \]

(b) The pole exerts a force on the beam of \(3.9 \times 10^2 \text{ N} \) at an angle of \(2.5^\circ\) up from the beam.

**Validate the Solution**

The tension in the cable is greater than the combined weights of the light and the beam \((27.5 \text{ kg} \times 9.81 \text{ m/s}^2 = 270 \text{ N})\), as it should be. The units give newtons as required. Based on the calculated tension in the cable, the value for the force exerted by the pole seems reasonable.

### 34. Frame the Problem

- Sketch the situation, indicating the directions of all the forces.
- Let the contact of the ladder with the ground be the axis of rotation.
- Use the equation for rotational equilibrium to find the normal force that the house exerts on the ladder. Note: it is assumed that the house is smooth, so at the point of contact between the ladder and the house, there is only the normal force.
- Use the equations for translational equilibrium in the \(x\) and \(y\) directions to find normal force at the ground and the friction force.
- Note that a common error in this problem is to get the direction of the force of friction wrong. Because the ladder wants to slip along the ground away from the house, the force of friction acts towards the house.

**Identify the Goal**

The friction force, \( \vec{F}_f \), of the ground on the base of the ladder to prevent it from slipping.

**Variables and Constants**

**Identify the Variables**

<table>
<thead>
<tr>
<th>Known</th>
<th>Implied</th>
<th>Unknown</th>
</tr>
</thead>
<tbody>
<tr>
<td>(m_1 = 55 \text{ kg})</td>
<td>(g = 9.81 \text{ m/s}^2)</td>
<td>(\vec{F}_f)</td>
</tr>
<tr>
<td>(m_2 = 7.5 \text{ kg})</td>
<td>(l_3 = 0.75 \times 2.6 \text{ m})</td>
<td>(\vec{F}_{\text{Nh}})</td>
</tr>
<tr>
<td>(l_1 = 2.6 \text{ m})</td>
<td>(= 1.95 \text{ m})</td>
<td>(\vec{F}_{\text{NG}})</td>
</tr>
<tr>
<td>(l_2 = 1.3 \text{ m})</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Develop a Strategy

Begin by writing the equation for rotational equilibrium.
Rearrange and isolate the tension force.
Note that the length of the beam cancels.
Consider the counterclockwise direction as positive.
Substitute numerical values and solve.
Apply the equations of translational equilibrium to find the force of friction.
First for the x-components.

Calculations

\[ \sum \tau = 0 \]
\[ \tau_{NG} + \tau_f + \tau_{gladder} + \tau_{gstudent} + \tau_{Nh} = 0 \]
\[ r_L F_{NG} + r_L F_f - r_L F_{gladder} - r_L F_{gstudent} + r_L F_{Nh} = 0 \]
\[ r_L F_{Nh} = -r_L F_{NG} - r_L F_f + r_L F_{gladder} + r_L F_{gstudent} \]
\[ F_{Nh} = \frac{-F_{Ng} - F_f + F_{gladder} + F_{gstudent}}{r_{LNh}} \]
\[ F_{Nh} = -0 F_{Ng} - F_f + r_L F_{gladder} + r_L F_{gstudent} \]
\[ F_{Nh} = \frac{r_L F_{gladder} + r_L F_{gstudent}}{r_{LNh}} \]
\[ F_{Nh} = \frac{(1.3 m) \cos 73^\circ (7.5 kg)(9.81 m/s^2) + (1.95 m) \cos 73^\circ (55 kg)(9.81 m/s^2)}{(2.6 m) \sin 73^\circ} \]
\[ F_{Nh} = 134.965 \text{ N} \]

\[ \sum F_x = 0 \]
\[ F_f - F_{Nh} = 0 \]
\[ F_f = F_{Nh} \]
\[ F_f = 134.965 \text{ N} \]
\[ F_f = 1.3 \times 10^5 \text{ N} \]
The force of friction must be $1.3 \times 10^2 \text{ N}$ to prevent the ladder from slipping.

Note the problem is now solved and we don't need the y-components of the forces.

**Validate the Solution**
The units worked out to be newtons in each case. The value for the friction seems reasonable.

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35. Conceptualize the Problem
- Sketch the vectors representing the momentum of the two billiard balls immediately before and just after the collision. It is always helpful to superimpose an $x$–$y$-coordinate system on the vectors so that the origin is at the point of contact of the two balls. For calculations, use the angles that the vectors make with the $x$-axis.
- Momentum is conserved in the $x$ and $y$ directions independently.
- The total momentum of the system (ball A and B) before the collision is carried by ball $A$ and is all in the positive $y$ direction.
- After the collision, both balls have momentum in both the $y$ direction and the $x$ direction.
- Since the momentum in the $x$ direction was zero before the collision, it must be zero after the collision. Therefore, the $x$-components of the momentum of the two balls after the collision must be equal in magnitude and opposite in direction.
- The sum of the $y$-components of the two balls after the collision must equal the momentum of billiard ball $A$ before the collision.
- As components of vectors are being considered, the calculations can be done in terms of magnitudes (instead of vectors).

**Identify the Goal**
The velocity, $\vec{v}_B'$, of ball B after the collision

<table>
<thead>
<tr>
<th>Known</th>
<th>Unknown</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_A = 0.150 \text{ kg}$</td>
<td>$\vec{v}_A' = 7.7 \text{ m/s}[40^\circ \text{ clockwise from } +y\text{-direction}] \text{ or } 7.7 \text{ m/s}$</td>
</tr>
<tr>
<td>$m_B = 0.150 \text{ kg}$</td>
<td>$\vec{v}_B'$</td>
</tr>
<tr>
<td>$\vec{v}_A = 10.0 \text{ m/s}[+y\text{-direction}]$</td>
<td>$[50^\circ \text{ counter-clockwise from the } +x\text{-direction}]$</td>
</tr>
<tr>
<td>$\vec{v}_B = 0 \text{ m/s}$</td>
<td></td>
</tr>
</tbody>
</table>

**Develop a Strategy**
Write the expression for the conservation of momentum in the $x$-direction.

Solve for the velocity of ball B after the collision.

The $x$-component of both balls before the collision was zero.

Substitute values and solve.

**Calculations**

\[
m_Av_Ax + m_Bv_Bx = m_Av_A'x + m_Bv_B'x
\]

\[
\vec{v}_B' = \frac{m_Av_Ax + m_Bv_Bx - m_Av_A'}{m_B}
\]

\[
\vec{v}_B' = \frac{0 + 0 - (0.150 \text{ kg})(7.7 \cos 50^\circ \text{ m/s})}{(0.150 \text{ kg})}
\]

\[
\vec{v}_B' = -4.9495 \text{ m/s}
\]
After the collision, ball B moves with a speed of 6.4 m/s in a direction 40.0° counter-clockwise from the +y-direction.

**Validate the Solution**

Since all of the momentum before the collision was in the positive y direction, the y-component of momentum after the collision had to be in the positive y direction, which it was. Since there was no momentum in the x direction before the collision, the x-components of the momentum after the collision had to be in opposite directions, which they were.

**36. Conceptualize the Problem**

- Sketch the vectors representing the momentum of the bowling ball and pin immediately before and just after the collision. It is always helpful to superimpose an x–y-coordinate system on the vectors so that the origin is at the point of contact of the two objects. For calculations, use the angles that the vectors make with the x-axis.
- Momentum is conserved in the x and y directions independently.
- The total momentum of the system (bowling ball A and pin B) before the collision is carried by the bowling ball (A) and is all in the positive y direction.
- After the collision, both the ball and pin have momentum in both the y direction and the x direction.
- Since the momentum in the x direction was zero before the collision, it must be zero after the collision. Therefore, the x-components of the momentum of the ball and pin after the collision must be equal in magnitude and opposite in direction.
- The sum of the y-components of the ball and pin after the collision must equal the momentum of bowling ball A before the collision.
- As components of vectors are being considered, the calculations can be done in terms of magnitudes (instead of vectors).
Identify the Goal
The velocity, $\overrightarrow{v}_B'$, of the bowling ball A after the collision

**Known**
- $m_A = 6.00$ kg
- $m_B = 0.220$ kg
- $\overrightarrow{v}_A = 1.20$ m/s [$+y$-direction]
- $\overrightarrow{v}_B = 0$ m/s

**Unknown**
- $\overrightarrow{v}_B' = 3.60$ m/s [$70.0^\circ$ counter-clockwise from $+y$-direction]
- $\overrightarrow{v}_A'$ or, $3.60$ m/s [$20.0^\circ$ clockwise from the $-x$-direction]

Develop a Strategy
Write the expression for the conservation of momentum in the $x$-direction.

Solve for the velocity of the bowling ball B after the collision.

The $x$-component of the system before the collision was zero.
After the collision, the pin travels in the negative $x$-direction.
Substitute values and solve.
Carry out the same procedure for the $y$-components.

Use the Pythagorean theorem to find the magnitude of the resultant velocity vector of the bowling ball (A).

Use the tangent function to find the direction of the velocity vector.

After the collision, the bowling ball (A) moves with a speed of 1.16 m/s in a direction $83.9^\circ$ clockwise from the $+x$-axis.

Validate the Solution
The bowling ball has a much greater mass than the pin so it is expected that its speed after the collision (1.16 m/s) will only be slightly less than before the collision (1.20 m/s), as is observed. Also, the path of the bowling ball is not expected
to change much due to the collision. After the collision, the bowling ball travels only 90.0 \(– 83.9 = 6.1^\circ\) from its original direction — a reasonable result.

37. Conceptualize the Problem
- Sketch a vector diagram of the momentum before and after the collision.
- Consider the two cars to be a "system." Before the collision, the south component of the momentum of the system was carried by car A and the east component was carried by car B.
- Momentum is conserved in the north-south direction and in the east-west direction independently.
- After the collision, the cars form one mass with all of the momentum.
- As components of vectors are being considered, the calculations can be done in terms of magnitudes (instead of vectors).

Identify the Goal
The velocities, \(\vec{v}_A'\) and \(\vec{v}_B'\), of the two cars before the collision

Identify the Variables
<table>
<thead>
<tr>
<th>Known</th>
<th>Unknown</th>
</tr>
</thead>
<tbody>
<tr>
<td>(m_A = 1750\text{ kg})</td>
<td>(\vec{v}_A)</td>
</tr>
<tr>
<td>(m_B = 1450\text{ kg})</td>
<td>(\vec{v}_B)</td>
</tr>
<tr>
<td>(\vec{v}_{AB} = 35.8 \text{ km/h}[E31.6^\circ S])</td>
<td>(\vec{v}_A')</td>
</tr>
</tbody>
</table>

Develop a Strategy
Write the equation for conservation of momentum.

Work with the north-south direction only. Modify the equation to show that car B was moving directly east before the crash; its north-south momentum was zero. After the crash, the cars were combined.

Solve the equation for the original velocity of car A.
Car A was travelling 34.3 km/h south and car B was travelling 67.3 km/h east at the instant before the crash.

Validate the Solution
The units all cancelled to give km/h, which is correct for velocity. After the collision, the cars travel together at an angle of less than 45˚, that is, more towards the east than towards the south [E31.6˚S]. Therefore, it is expected that the momentum of car B, which is travelling east before the collision, will be greater than the momentum of car A. Before the collision, car A’s momentum is \( \vec{m}_A v_A [S] = (1750 \text{ kg})(34.3 \text{ km/h}) = 6.0 \times 10^4 \text{ kg km/h} [S] \), and car B’s momentum is \( \vec{m}_B v_B [E] = (1450 \text{ kg})(67.3 \text{ km/h}) = 9.8 \times 10^4 \text{ kg km/h} [E] \), so, the eastward momentum before the collision is greater than the southward momentum, as expected.

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38. Frame the Problem
- Make a sketch of the momentum vectors after the explosion.
- After an explosion, the vector sum of the momentum of all the fragments must be zero.

Identify the Goal
The mass, \( m_3 \), and velocity, \( \vec{v}_3 \), of the third fragment after the explosion.
Identify the Variables

**Known**

- \( m_1 \) = 1.3 kg
- \( m_2 \) = 1.2 kg
- \( \vec{v}_1 \) = 1.8 m/s
- \( \theta_1 \) = 52° ccw from +x-axis
- \( \vec{v}_2 \) = 1.8 m/s
- \( \theta_2 \) = 61° cw from −x-axis

**Unknown**

- \( m_3 \)
- \( \vec{v}_3 \)

Develop a Strategy

**Calculations**

Find the \( x \)- and \( y \)-components of the momentum of fragments 1 and 2.

Because fragment 2 is in the 3rd quadrant, both its components are negative.

Sum the \( x \)- and \( y \)-components of the momentum individually.

Begin with the \( x \)-components.

Then repeat for the \( y \)-components.

Use the Pythagorean theorem to find the magnitude of the momentum of the third fragment.

Use the tangent function to find the direction of the momentum of fragment 3.

Find the mass of the third fragment from the total mass.

\[
\begin{align*}
\text{Find the mass of the third fragment from the total mass:} \\
m_1 + m_2 + m_3 &= 3.5 \text{ kg} \\
m_3 &= 3.5 \text{ kg} - 1.3 \text{ kg} - 1.2 \text{ kg} \\
m_3 &= 1.0 \text{ kg}
\end{align*}
\]
Find the velocity of fragment 3 from the definition of momentum. 

\[ \vec{p} = m \vec{v} \]

\[ |\vec{v}| = \frac{|\vec{p}|}{m} \]

\[ |\vec{v}| = \frac{4.5 \text{ kg m/s}}{1.0 \text{ kg}} \]

\[ |\vec{v}| = 4.5 \text{ m/s} \]

The velocity of the third fragment was 4.5 m/s at an angle of 89.8° ccw from the +x-axis.

**Validate the Solution**

The units worked properly in each case. The third fragment has the smallest mass of the three fragments, so it is expected to have a large velocity, and it does. The value seems reasonable.

**Practice Problem Solutions**

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39. **Conceptualize the Problem**

- Sketch the vectors representing the momentum of the two billiard balls immediately before and just after the collision. Superimpose an \( x-y \)-coordinate system on the vectors so that the origin is at the point of contact of the two balls. For calculations, use the angles that the vectors make with the \( x \)-axis.
- Momentum is conserved in the \( x \) and \( y \) directions independently.
- The total momentum of the system (ball A and B) before the collision is carried by ball A and is all in the positive \( y \) direction.
- After the collision, both balls have momentum in both the \( y \) direction and the \( x \) direction.
- Since the momentum in the \( x \) direction was zero before the collision, it must be zero after the collision. Therefore, the \( x \)-components of the momentum of the two balls after the collision must be equal in magnitude and opposite in direction.
- The sum of the \( y \)-components of the two balls after the collision must equal the momentum of billiard ball A before the collision.
- Total momentum is always conserved in a collision.
- If the collision is elastic, total kinetic energy must also be conserved.
- As components of vectors are being considered, the calculations can be done in terms of magnitudes (instead of vectors).
Identify the Goal
Whether the total kinetic energy of the system before the collision, $E_{kA}$, is equal to the total kinetic energy of the system after the collision, $E'_{kA} + E'_{kB}$

Identify the Variables

<table>
<thead>
<tr>
<th>Known</th>
<th>Unknown</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_A = 0.155 \text{ kg}$</td>
<td>$v'_A = 9.56 \text{ m/s}[29.7^\circ$ clockwise from $+y$-direction]</td>
</tr>
<tr>
<td>$m_B = 0.155 \text{ kg}$</td>
<td>$E_{kB} = 0 \text{ J}$</td>
</tr>
<tr>
<td>$v_A = 12.5 \text{ m/s}[+y$-direction]</td>
<td>$= 9.56 \text{ m/s}[60.3^\circ$ counter-clockwise from $+x$-direction]</td>
</tr>
<tr>
<td>$v_B = 0 \text{ m/s}$</td>
<td></td>
</tr>
</tbody>
</table>

Develop a Strategy

Write the expression for the conservation of momentum in the $x$-direction.

Solve for the velocity of ball B after the collision.

The $x$-component of both balls before the collision was zero. Notice the masses divide out.

Substitute values and solve.

Carry out the same procedure for the $y$-components. Notice the masses divide out.

Use the Pythagorean theorem to find the magnitude of the resultant velocity vector of ball B.

Use the tangent function to find the direction of the velocity vector.

After the collision, ball B moves with a speed of 6.3 m/s in a direction 41.5° clockwise from the $-x$-direction.
Calculate the kinetic energy of ball A before the collision.
\[ E_{kA} = \frac{1}{2} m_A v_A^2 \]
\[ E_{kA} = \frac{1}{2} (0.155 \text{ kg}) \left( 12.5 \text{ m/s} \right)^2 \]
\[ E_{kA} = 12.1 \text{ J} \]

Calculate the sum of the kinetic energies of the balls after the collision.
\[ E'_k = \frac{1}{2} m_A v'_A^2 + \frac{1}{2} m_B v'_B^2 \]
\[ E'_k = \frac{1}{2} (0.155 \text{ kg}) \left( 9.56 \text{ m/s} \right)^2 + \frac{1}{2} (0.155 \text{ kg}) \left( 6.33 \text{ m/s} \right)^2 \]
\[ E'_k = 10.2 \text{ J} \]

The kinetic energy before the collision is greater than the kinetic energy after the collision by almost 2.0 J, so the collision was not elastic.

**Validate the Solution**

After the collision, the stationary billiard ball travels away at a larger angle (41.5˚ to the left) than the moving billiard ball (29.7˚ to the right). Because it is a "glancing" collision, it is reasonable that energy could be dissipated during the collision so that the kinetic energy after the collision is less than the kinetic energy before.

**40. Conceptualize the Problem**

- Sketch the vectors representing the momentum of the two cars immediately before and just after the collision.
- Consider the two cars to be a “system.” Before the collision, the north component of the momentum of the system was carried by car A and the west component was carried by car B.
- Momentum is conserved in the north-south direction and in the east-west direction independently.
- After the collision, the cars form one mass with all of the momentum.
- Total momentum is always conserved in a collision.
- If the collision is elastic, total kinetic energy must also be conserved.

\[ \vec{p}_{\text{AB}}' = ? \]
\[ \vec{v}_B = 37.7 \text{ km/h[W]} \]
\[ \vec{v}_A = 45.5 \text{ km/h[N]} \]

**Identify the Goal**

The total momentum of the system, \( \vec{p}_{\text{AB}}' \), after the collision

Whether the total kinetic energy of the system before the collision, \( E_{kA} + E_{kB} \), is equal to the total kinetic energy of the system after the collision, \( E'_{kA} + E'_{kB} \)
Identify the Variables

<table>
<thead>
<tr>
<th>Known</th>
<th>Unknown</th>
</tr>
</thead>
<tbody>
<tr>
<td>( m_A = 1735 \text{ kg} )</td>
<td>( \vec{v}'_{AB} )</td>
</tr>
<tr>
<td>( m_B = 2540 \text{ kg} )</td>
<td>( P_{AB} )</td>
</tr>
<tr>
<td>( \vec{v}_A = 45.5 \text{ km/h}[\text{N}] )</td>
<td>( E_{kA} )</td>
</tr>
<tr>
<td>( \vec{v}_B = 37.7 \text{ km/h}[\text{W}] )</td>
<td>( E_{kB} )</td>
</tr>
</tbody>
</table>

Develop a Strategy

Write the expression for the conservation of momentum in the N–S–direction.

\[ m_A \vec{v}_A[N] + m_B \vec{v}_B[N] = (m_A + m_B) \vec{v}'_{AB}[N] \sin \theta \]

Notice \( \theta \) is the angle measured clockwise from the negative \( x \)-axis (the west direction).

The initial momentum of car B in this direction is zero.

Write the expression for the conservation of momentum in the E–W–direction.

\[ m_A \vec{v}_A[W] + m_B \vec{v}_B[W] = (m_A + m_B) \vec{v}'_{AB}[W] \cos \theta \]

The initial momentum of car A in this direction is zero.

There are now two equations with two unknowns, the final combined velocity of the cars, \( \vec{v}'_{AB} \) and the direction they travel, \( \theta \).

Divide equation (1) by equation (2) and solve for the angle.

\[ \frac{m_A \vec{v}_A[N]}{m_B \vec{v}_B[W]} = \frac{(m_A + m_B) \vec{v}'_{AB}[N] \sin \theta}{(m_A + m_B) \vec{v}'_{AB}[W] \cos \theta} \]

\[ \frac{m_A \vec{v}_A[N]}{m_B \vec{v}_B[W]} = \frac{\sin \theta}{\cos \theta} = \tan \theta \]

The directions, combined masses and velocity cancels from the equation. The trigonometry terms simplify.

The directions, combined masses and velocity cancels from the equation. The trigonometry terms simplify.

\[ \tan \theta = \frac{(1735 \text{ kg})(45.5 \text{ km/h})}{(2540 \text{ kg})(37.7 \text{ km/h})} \]

\[ \theta = \tan^{-1}(0.82440) \]

\[ \theta = 39.502^\circ \]

\[ \theta \approx 39.5^\circ \]

Substitute known values and solve. The cars move together after the collision in a direction W39.5°N. Use this result in equation (1).

Substitute known values and solve.

\[ m_A \vec{v}_A[N] = (m_A + m_B) \vec{v}'_{AB}[N] \sin \theta \]

\[ \vec{v}'_{AB} = \frac{m_A \vec{v}_A}{(m_A + m_B) \sin \theta} \]

\[ \vec{v}'_{AB} = \frac{(1735 \text{ kg})(45.5 \text{ km/h})}{(1735 \text{ kg} + 2540 \text{ kg}) \sin (39.502^\circ)} \]

\[ \vec{v}'_{AB} = 29.03 \text{ km/h} \]

Determine the momentum of the combined cars immediately after the collision.

\[ \vec{P}_{AB} = m_A \vec{v}_A \]

\[ \vec{P}_{AB} = (1735 \text{ kg} + 2540 \text{ kg})(29.03 \text{ km/h})[\text{W39.5°N}] \]

\[ \vec{P}_{AB} = 1.2410 \times 10^5 \text{ kg km/h}[\text{W39.5°N}] \]

\[ \vec{P}_{AB} \equiv 1.24 \times 10^5 \text{ kg km/h}[\text{W39.5°}] \]
The kinetic energy of the system before the collision is a factor of two greater than the kinetic energy after the system, so the collision is not elastic.

Validate the Solution
The speed of the combined cars after the collision, 29.0 km/h, is lower than the speed of either car before the collision, as expected. Also, as this is a collision between cars, it is expected that some kinetic energy will be lost to deforming the cars during the collision, and it is not expected to be elastic, as is the case.

Practice Problem Solutions
Student Textbook pages 524–525

41. Conceptualize the Problem
- Sketch the positions of the bullet and pendulum bob just before the collision, just after the collision, and with the pendulum at its highest point.
- When the bullet hit the pendulum, total momentum was conserved.
- If you can find the velocity of the combined bullet and pendulum bob after the collision, you can use conservation of momentum to find the velocity of the bullet before the collision.
- The collision was completely inelastic so total kinetic energy was not conserved.
- However, you can assume that the friction of the pendulum is negligible, so mechanical energy of the pendulum was conserved.
- The gravitational potential energy of the combined masses at the highest point of the pendulum is equal to the kinetic energy of the combined masses at the lowest point of the pendulum.
- If you know the kinetic energy of the combined masses just after the collision, you can find the velocity of the masses just after the collision.
- Use the subscripts "b" for bullet and "p" for pendulum.

Identify the Goal
The velocity, \( v_b \), of the bullet just before it hit the ballistic pendulum

Identify the Variables

<table>
<thead>
<tr>
<th>Known</th>
<th>Implied</th>
<th>Unknown</th>
</tr>
</thead>
<tbody>
<tr>
<td>( m_b = 12.5 \text{ g} )</td>
<td>( m = \frac{9.81 \text{ m}}{s^2} )</td>
<td>( v_b )</td>
</tr>
<tr>
<td>( m_p = 2.37 \text{ kg} )</td>
<td></td>
<td>( E_g )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( v_p )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( E_k )</td>
</tr>
</tbody>
</table>
Develop a Strategy

To find the velocity of the combined masses of the bullet and pendulum bob just after the collision, use the relationship that describes the conservation of mechanical energy of the pendulum.

Substitute the expressions for kinetic energy and gravitational potential energy known from previous physics courses. Solve for velocity. Convert all units to SI units.

Define the direction of the bullet as positive during and immediately after the collision.

Apply the conservation of momentum to find the velocity of the bullet before the collision. Convert all units to SI units. Motion is in a single dimension so magnitudes can be used in the calculation.

The speed of the bullet just before the collision was about 261 m/s in the positive direction.

Validate the Solution

In both calculations, the units cancelled to give metres per second, which is correct for velocity. A speed of 261 m/s for a bullet is a reasonable result.

42. Conceptualize the Problem

- Sketch the positions of the ball of putty and the pendulum bob just before the collision, just after the collision, and with the cart at its highest point on the track.
- When the putty hit the cart, total momentum was conserved.
- The velocity of the putty before the collision is known, so conservation of momentum can be used to find the velocity of the putty–cart system immediately after impact.
- The collision was completely inelastic so total kinetic energy was not conserved.
- However, because the cart ascended an air track, it can be assumed that friction can be ignored, so mechanical energy of the putty–cart system was conserved.
- The kinetic energy of the putty–cart system at its maximum height up the track is zero.
- The gravitational potential energy of the combined masses at the highest point of the air track is equal to the kinetic energy of the combined masses at the lowest point of
the air track (that is, before the cart begins its ascent up the track, when the putty hits the cart).

- The gravitational potential energy gives the height up the track that the cart ascends. From this, the distance the putty–cart system travels up the track can be calculated from trigonometry.

![Diagram](image)

**Identify the Goal**
The distance, \( \Delta x \), the putty–cart system travels up the track (and whether it travels to the end)

**Identify the Variables**

<table>
<thead>
<tr>
<th>Known</th>
<th>Implied</th>
<th>Unknown</th>
</tr>
</thead>
<tbody>
<tr>
<td>( m_p = 23 \text{ g} )</td>
<td>( g = 9.81 \text{ m/s}^2 )</td>
<td>( v'_{pc} )</td>
</tr>
<tr>
<td>( m_c = 225 \text{ g} )</td>
<td></td>
<td>( h_f )</td>
</tr>
<tr>
<td>( d = 1.5 \text{ m} )</td>
<td></td>
<td>( \Delta x )</td>
</tr>
<tr>
<td>( \theta = 25^\circ )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Develop a Strategy**

Apply the conservation of momentum to find the velocity of the putty–cart system immediately after the collision. Convert all units to SI units.

Before the cart ascends the ramp, all the motion is in one dimension, so magnitudes can be used in the calculation.

To find the height that the combined masses travel up the track after the collision, use the relationship that describes the conservation of mechanical energy of the system.

The velocity of the combined putty–cart system is the velocity at the bottom of the track.

Solve for the final height that the system travels up the track.
The system rolls about 0.018 m along the track and thus, it stops far short of the end.

**Validate the Solution**
The ball of putty has a low mass (23 g) so it is not expected to give the cart, whose mass is ten times greater, a large velocity. The velocity of the putty–cart system is only about 0.4 m/s, which seems about right. This is a low velocity, so it is not expected that the system will travel very far up the sloped track. A distance of 0.018 m = 1.8 cm seems reasonable. In the calculation of the final height, the units worked out to be metres, as required.

43. **Conceptualize the Problem**
- Sketch the cars just before, at the moment of, and after the collision, when they came to a stop.
- Total momentum is conserved during the collision. However, momentum conservation cannot be applied first because there are two unknowns (the velocity of the second car before the collision, and the combined velocity after the collision). It must be applied after the combined velocity of the two cars is found.
- Because the cars stuck together, the collision was completely inelastic, so total kinetic energy was not conserved. Some kinetic energy was lost to sound, heat, and deformation of the metal during the collision.
- Some kinetic energy remained after the collision.
- The force of friction did work on the moving cars, converting the remaining kinetic energy into heat.
- From the law of conservation of energy, the work done by the force of friction is equal to the kinetic energy of the cars at the instant after the collision.
- Since the motion is in one direction, use a plus sign to symbolize that direction.
- Let car A be the stopped car and car B be the moving car.

**Identify the Goal**
The speed, $v_B$, of the second car before the collision

**Identify the Variables**

<table>
<thead>
<tr>
<th>Known</th>
<th>Implied</th>
<th>Unknown</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_A = 1875$ kg</td>
<td>$v_A = 0$ m/s</td>
<td>$g = 9.81$ m/s$^2$</td>
</tr>
<tr>
<td>$m_B = 2135$ kg</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\mu = 0.750$</td>
<td>$\Delta d = 4.58$ m</td>
<td></td>
</tr>
</tbody>
</table>

**Develop a Strategy**
Due to the law of conservation of energy, the work done on the cars by the force of friction is equal to the kinetic energy of the connected cars after the collision.

**Calculations**

$W_{\text{(to stop cars)}} = E_k$ (immediately after collision)

Use trigonometry to determine the distance the cart travels along the track. Let $\Delta x$ be the distance along the slanted track (i.e. the hypotenuse of the right triangle).

The sine function applies. Solve for $\Delta x$.

$\Delta x = \frac{0.007732 \, \text{m}}{\sin 25^\circ} = 0.0183 \, \text{m}$

$\Delta x \approx 0.018 \, \text{m}$
Substitute the expressions for work and kinetic energy into the equations. 

\[ F\Delta d = \frac{1}{2} m_{AB} v_{AB}^2 \]

Both quantities (work and kinetic energy) are scalars, so only the magnitudes of the displacement and velocity are needed.

Friction is the force doing the work, and it is always parallel to the direction of motion.

Substitute the formula for the force of friction.

\[ F = \mu N \]

Since the cars are moving horizontally, the normal force is the weight of the cars. Substitute the weight into the expression and solve for the velocity.

Apply the conservation of momentum to find the velocity of the second car (car B) immediately before the collision.

As the motion is in one dimension, use magnitudes in the calculation.

\[ m_A v_A + m_B v_B = (m_A + m_B) v_{AB}' \]

\[ 0 + m_B v_B = (m_A + m_B) v_{AB}' \]

\[ v_{AB}' = \frac{(1875 \text{ kg} + 2135 \text{ kg})(8.209 \text{ kg})}{2135 \text{ kg}} \]

\[ v_{AB}' = 8.209 \text{ m/s} \]

The speed of the second car before the collision was about 15.4 m/s.

**Validate the Solution**

In the work-kinetic energy calculation, the units cancelled to give m/s, as required for velocity. The velocity of car B before the collision, 15.4 m/s, is about 55.5 km/h, which is a reasonable highway speed.

**44. Conceptualize the Problem**

- Sketch the positions of the ball and pendulum box just before the collision, just after the collision, and with the pendulum at its highest point.
- When the ball hit the pendulum, total momentum was conserved.
- If you can find the velocity of the combined ball and pendulum box after the collision, you can use conservation of momentum to find the velocity of the ball before the collision.
- The collision was completely inelastic so total kinetic energy was not conserved.
- However, you can assume that the friction of the pendulum is negligible, so mechanical energy of the pendulum was conserved.
- The gravitational potential energy of the combined masses at the highest point of the pendulum is equal to the kinetic energy of the combined masses at the lowest point of the pendulum.
- The highest point of the pendulum can be determined using the measured angle and trigonometry.
- If you know the kinetic energy of the combined masses just after the collision, you can find the velocity of the masses just after the collision.
- Use the subscripts "b" for ball and "p" for pendulum.

Identify the Goal
The velocity, \( v_b \), of the ball just before it hit the pendulum box.

Identify the Variables

<table>
<thead>
<tr>
<th>Known</th>
<th>Implied</th>
<th>Unknown</th>
</tr>
</thead>
<tbody>
<tr>
<td>( m_b = 0.350 \text{ kg} )</td>
<td></td>
<td>( v_b )</td>
</tr>
<tr>
<td>( m_p = 5.64 \text{ kg} )</td>
<td>( g = 9.81 \frac{m}{s^2} )</td>
<td>( v_{bp} )</td>
</tr>
<tr>
<td>( \theta = 20.0^\circ )</td>
<td></td>
<td>( \Delta h )</td>
</tr>
<tr>
<td>( L = 0.955 \text{ m} )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Develop a Strategy
To find the velocity of the combined masses of the ball and pendulum box just after the collision, use the relationship that describes the conservation of mechanical energy of the pendulum.

Substitute the expressions for kinetic energy and gravitational potential energy known from previous physics courses. Solve for velocity. The height of the box is determined from trigonometry:

\[ \Delta h = L - L \cos \theta \]

= \( L(1 - \cos \theta) \). Define the direction of the ball as positive during and immediately after the collision.

\[ E_{k(bottom)} = E_{g(top)} \]

\[ \frac{1}{2} m v_{bottom}^2 = mg\Delta h \]

\[ v_{bottom}^2 = 2g\Delta h \]

\[ v_{bottom} = \sqrt{2g\Delta h} \]

\[ v_{bottom} = \sqrt{2gL(1 - \cos \theta)} \]

\[ v_{bottom} = \pm 1.06301 \frac{m}{s} \]

Calculations

\[ h = L - L \cos \theta \]
The speed of the ball just before the collision was about 18.2 m/s in the positive direction.

**Validate the Solution**

The angle of deflection, 20.0°, is relatively small, so the velocity of the ball is not expected to rival that of a major league pitcher. A value of 18.2 m/s seems reasonable. Note that a deflection angle of greater than 52° would be required for the speed of the pitch to be greater than 46 m/s.

45. **Conceptualize the Problem**

- Sketch the position of the boulder when it is on the side of the mountain, just before the collision with the shack, and just after the collision.
- The **gravitational potential energy** of the boulder on the side of the mountain is equal to the **kinetic energy** of the boulder immediately before the collision.
- When the boulder hit the shack, total **momentum was conserved**.
- Because you can find the **velocity** of the boulder before the collision, you can use conservation of momentum to find the velocity of the boulder–shack system immediately after the collision.
- The collision was completely inelastic (the boulder and shack stuck together) so **kinetic energy was not conserved**. Some kinetic energy was lost to sound, heat, and deformation of the shack during the collision.
- Some **kinetic energy** remained after the collision.
- The **force of friction** did work on the boulder–shack system, converting the remaining kinetic energy to heat.
- From the **law of conservation of energy**, the work done by the **force of friction** is equal to the **kinetic energy** of the system at the instant after the collision.
- Since the motion is in one direction, use a plus sign to symbolize direction.

**Identify the Goal**

The velocity, \( v_{bo} \), of the boulder–shack system at the instant they began to slide across the ice, and the distance, \( \Delta d \), that they slide across the ice

**Identify the Variables**

<table>
<thead>
<tr>
<th>Known</th>
<th>Implied</th>
<th>Unknown</th>
</tr>
</thead>
<tbody>
<tr>
<td>( m_b = 55.6 \text{ kg} )</td>
<td>( g = 9.81 \text{ m/s}^2 )</td>
<td>( v_b )</td>
</tr>
<tr>
<td>( m_p = 204 \text{ kg} )</td>
<td></td>
<td>( v_b' )</td>
</tr>
<tr>
<td>( \Delta h = 14.6 \text{ m} )</td>
<td></td>
<td>( \Delta d )</td>
</tr>
<tr>
<td>( \mu = 0.392 )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Develop a Strategy

To find the velocity of the combined masses of the boulder and shack just after the collision, use the relationship that describes the conservation of mechanical energy of the boulder.

Substitute the expressions for kinetic energy and gravitational potential energy known from previous physics courses. Solve for velocity.

Define the direction of the boulder as positive during and immediately after the collision.

Apply the conservation of momentum to find the velocity of the boulder–shack system immediately after the collision.

The velocity of the boulder–shack system immediately after the collision is about 3.62 m/s [forward].

Due to the law of conservation of energy, the work done on the boulder–shack system by the force of friction is equal to the kinetic energy of the system after the collision.

Substitute the expressions for work and kinetic energy into the equations. Both quantities (work and kinetic energy) are scalars, so only the magnitudes of the displacement and velocity are needed.

Friction is the force doing the work, and it is always parallel to the direction of motion. Substitute the formula for the force of friction.
The shack and boulder slide about 1.71 m across the ice.

**Validate the Solution**
The units for the distance are metres, as required. The shack is nudged only 1.7 m by the boulder, which seems reasonable.

### Chapter 10 Review

**Answers to Problems for Understanding**

Student Textbook pages 180–181

23. (a) The kayak should be pointed 54° upstream. The unknown velocity vector is the hypotenuse of a triangle whose other sides are known.

\[
\sin \theta = \frac{|v_{ws}|}{|v_{kw}|}
\]

\[
\theta = \sin^{-1} \frac{2.1 \text{ m/s}}{2.6 \text{ m/s}}
\]

\[
\theta = 53.87°
\]

\[
\theta \approx 54°
\]

(b) The kayak’s velocity relative to the shore will be 1.5 m/s. Two of the three sides of a right triangle are known. Use the Pythagorean theorem to determine the third side.

\[
|v_{kw}|^2 + |v_{ws}|^2 = |v_{kw}|^2
\]

\[
|v_{kw}|^2 = |v_{kw}|^2 - |v_{ws}|^2
\]

\[
|v_{kw}| = (2.6 \text{ m/s})^2 - (2.1 \text{ m/s})^2
\]

\[
|v_{kw}| = 2.35 \text{ (m/s)}^2
\]

\[
|v_{kw}| = 1.533 \text{ m/s}
\]

\[
|v_{kw}| \approx 1.5 \text{ m/s}
\]

(c) It will take 29 s to paddle across. Use the definition of velocity,

\[
v_{ks} = \frac{\Delta \vec{d}}{\Delta t}
\]

\[
\Delta t = \frac{\Delta \vec{d}}{v_{ks}}
\]

\[
\Delta t = \frac{45 \text{ m}}{1.533 \text{ m/s}} = 29.35 \text{ s}
\]

\[
\Delta t \approx 29 \text{ s}
\]
24. (a) The fourth vector would be $1.0 \times 10^2$ N[E$27^\circ$S].
The problem is done exactly like practice problem number 7.
(b) The fourth vector would be $34$ N[E$89^\circ$S].
The problem is done exactly like practice problem number 7.
(c) The fifth force would be $1.4 \times 10^2$ N[E$66.8^\circ$N].
The problem is done exactly like practice problem number 7.

25. The coefficient of kinetic friction would be 0.16.
Find the net force of the three dogs in the forward $x$-direction.

$$F_{1x} = (83 \text{ N})\cos15.5^\circ = 79.98 \text{ N}$$
$$F_{2x} = (75 \text{ N})\cos9.0^\circ = 74.08 \text{ N}$$
$$F_{3x} = (77 \text{ N})\cos12.0^\circ = 75.32 \text{ N}$$

The $x$-component of the forward force is

$$F_{\text{dogn}} = 79.98 \text{ N} + 93.83 \text{ N} + 75.317 \text{ N} = 249.13 \text{ N}$$

Calculate the $y$-component:

$$F_{1y} = (83 \text{ N})\sin15.5^\circ = 22.18 \text{ N}$$
$$F_{2y} = (75 \text{ N})\sin9.0^\circ = -11.732 \text{ N}$$
$$F_{3y} = (77 \text{ N})\sin12.0^\circ = -16.009 \text{ N}$$

The $y$-component of the forward force is

$$F_{\text{dogny}} = 22.18 \text{ N} - 11.732 \text{ N} - 16.009 \text{ N} = 5.561 \text{ N}$$

Apply Newton's second law. Since the sled has constant velocity, it's acceleration is zero. The only force opposing the force of the dogs, is the force of friction.

$$F_{\text{dogn}} - F_f = ma_x = 0$$
$$F_{\text{dogn}} = F_f$$
$$F_{\text{dogn}} = \mu_k F_N$$
$$F_{\text{dogn}} = \mu_k mg$$
$$\mu_k = \frac{F_{\text{dogn}}}{mg}$$
$$\mu_k = \frac{229.44 \text{ N}}{(145 \text{ kg})(9.81 \text{ m} / \text{s}^2)}$$
$$\mu_k = 0.16$$

26. No, they will not move.
(Note: the free body diagrams for this question are the same as for Practice Problem #27.)
First, find the forces parallel to the plane. The force of gravity for mass 2 acts opposite to the combination of the force of friction and the parallel component of the force of gravity for the block.
To find the frictional force on $m_1$, apply Newton’s second law in the direction perpendicular to the plane.

\[ F_{\perp} = ma_{\perp} = 0 \]
\[ F_N - F_{g1\perp} = 0 \]
\[ F_N = m_1g \cos \theta \]

\[ F_f = \mu_s F_N \]
\[ F_f = \mu_s m_1g \cos \theta \]

\[ F_f = (0.42)(47 \text{ kg})(9.81 \text{ m/s}^2) \cos 25^\circ \]
\[ F_f = 175.50 \text{ N} \]

\[ F_{g1} = m_1g \sin \theta \]
\[ F_{g1} = (47 \text{ kg})(9.81 \text{ m/s}^2) \sin 25^\circ \]
\[ F_{g1} = 194.86 \text{ N} \]

\[ F_{g2} = m_2g \]
\[ F_{g2} = (35 \text{ kg})(9.81 \text{ m/s}^2) \]
\[ F_{g2} = 343.35 \text{ N} \]

\[ F_{g1} + F_f = (194.86 \text{ N}) + (175.50 \text{ N}) = 370.35 \]

Since $F_{g1} + F_f > F_{g2}$, the masses will not move.

(b) N/A

(c) You need to add 2.8 kg to $m_2$ to cause the masses to begin to move.

Set $F_{g2} = 370.35 \text{ N}$, i.e. the sum of $F_{g1}$ and $F_f$, and determine the mass that this force implies.

\[ F_{g2} = m_2'g = 370.35 \text{ N} \]
\[ m_2' = \frac{370.35 \text{ N}}{9.81 \text{ m/s}^2} = 37.75 \text{ kg} \]

Now determine the difference between this mass and the given mass:
\[ \Delta m = m_2' - m_2 = 37.75 \text{ kg} - 35 \text{ kg} = 2.75 \text{ kg} \]
\[ \Delta m \equiv 2.8 \text{ kg} \]

(d) The acceleration of the masses would be 1.1 m/s$^2$.

Apply Newton’s law in the $x$-direction (parallel to the plane):

\[ F_x = ma_x \]
\[ F_{g2x} - F_{g1x} - F_f = (m_1 + m_2)a_x \]
\[ m_2g - m_1g \sin \theta - \mu_s F_N = (m_1 + m_2)a_x \]
\[ a_x = \frac{m_2g - m_1g \sin \theta - \mu_s (m_1g \sin \theta)}{(m_1 + m_2)} \]
\[ a_x = \frac{m_2 - m_1 \sin \theta - \mu_s m_1 \sin \theta}{(m_1 + m_2)} \]
\[ a_x = \frac{37.75 \text{ kg} - (47 \text{ kg}) \sin 25^\circ - (0.19)(47 \text{ kg}) \cos 25^\circ}{(47 \text{ kg} + 37.75 \text{ kg})} \times 9.81 \text{ m/s}^2 \]
\[ a_x = 1.134 \text{ m/s}^2 \]
\[ a_x \equiv 1.1 \text{ m/s}^2 \]
27. The left side of the crevasse is pushing up on the ladder with $F_{c1} = 3.9 \times 10^2$ N and the right side of the crevasse is pushing up on the ladder with $F_{c2} = 5.0 \times 10^2$ N.

Consider the far side of the crevasse, c1, as the axis.

$\Sigma \tau = 0$

$- \frac{l}{2} F_i - l_{mc} F_{mc} + l F_{c2} = 0$

$F_{c2} = \frac{\frac{l}{2} F_i + l_{mc} F_{mc}}{l}$

$F_{c2} = \frac{\frac{41}{2} m (3.6 \text{ kg})(9.81 \text{ m} / \text{s}^2) + (1.8 \text{ m})(87 \text{ kg})(9.81 \text{ m} / \text{s}^2)}{4.1 \text{ m}}$

$F_{c2} = 392.35 \text{ N}$

$F_{c2} \cong 3.9 \times 10^2 \text{ N}$

$\Sigma F = 0$

$F_{c1} + F_{c2} - F_{mc} - F_i = 0$

$F_{c1} = F_{mc} + F_i - F_{c2}$

$F_{c1} = m_{mc} g + m_i g - F_{c2}$

$F_{c1} = (87 \text{ kg})(9.81 \text{ m} / \text{s}^2) + (3.6 \text{ kg})(9.81 \text{ m} / \text{s}^2) - 392.35 \text{ N}$

$F_{c1} = 496.44 \text{ N}$

$F_{c1} \cong 5.0 \times 10^2 \text{ N}$

28. crane
(a) The tension in the arm is 8.58 \times 10^3 N. 
\[ \Sigma \tau = 0 \text{ (choose the intersection of the beam with the crane pole as the axis)} \]

\[ l_{\text{beam}}F_T \sin \theta - l_2F_{\text{container}} - \frac{l_{\text{beam}}}{2}F_{\text{beam}} = 0 \]

\[ l_{\text{beam}}F_T \sin \theta = l_2F_{\text{container}} + \frac{l_{\text{beam}}}{2}F_{\text{beam}} \]

\[ F_T = \frac{l_2F_{\text{container}} + \frac{l_{\text{beam}}}{2}F_{\text{beam}}}{l_{\text{beam}} \sin \theta} \]

\[ F_T = \frac{(6.15 \text{ m})(355 \text{ kg})(9.81 \text{ m} / \text{s}^2) + \left(\frac{7.50 \text{ m}}{2}\right)(345 \text{ kg})(9.81 \text{ m} / \text{s}^2)}{(7.50 \text{ m}) \sin 32^\circ} \]

\[ F_T = 8582.28 \text{ N} \]

\[ F_T \cong 8.58 \times 10^3 \text{ N} \]

(b) The crane exerts a force of 7.64 \times 10^3 \text{ N} [17.7^\circ \text{ up from the beam}].

\[ \Sigma F_x = 0 \]

\[ F_{\text{cranex}} - F_{\text{Tx}} = 0 \]

\[ F_{\text{cranex}} = F_T \cos 32^\circ \]

\[ F_{\text{cranex}} = (8582.28 \text{ N}) \cos 32^\circ \]

\[ F_{\text{cranex}} = 7278.19 \text{ N} \]

\[ \Sigma F_y = 0 \]

\[ F_{\text{craney}} - F_{\text{beam}} - F_{\text{container}} + F_{\text{Ty}} = 0 \]

\[ F_{\text{craney}} = F_{\text{beam}} + F_{\text{container}} - F_{\text{Ty}} \]

\[ F_{\text{craney}} = (345 \text{ kg})(9.81 \text{ m} / \text{s}^2) + (355 \text{ kg})(9.81 \text{ m} / \text{s}^2) - (8582.28 \text{ N}) \sin 32^\circ \]

\[ F_{\text{craney}} = 2319.08 \text{ N} \]

Apply the Pythagorean theorem.

\[ |\vec{F}_{\text{craney}}|^2 = F_{\text{cranex}}^2 + F_{\text{craney}}^2 \]

\[ |\vec{F}_{\text{craney}}|^2 = (7278.19 \text{ N})^2 + (2319.08 \text{ N})^2 \]

\[ |\vec{F}_{\text{craney}}|^2 = 58350181.7 \text{ N}^2 \]

\[ |\vec{F}_{\text{craney}}| = 7638.73 \text{ N} \]

\[ |\vec{F}_{\text{craney}}| \cong 7.64 \times 10^3 \text{ N} \]

Apply the tangent function:

\[ \tan \theta = \frac{F_{\text{craney}}}{F_{\text{cranex}}} \]

\[ \theta = \tan^{-1} \frac{2319.08 \text{ N}}{7278.19 \text{ N}} \]

\[ \theta = \tan^{-1} 0.318634 \]

\[ \theta = 17.674^\circ \]

\[ \theta \cong 17.7^\circ \]
29. Velocity and direction of the eight ball: Assume that “forward,” the original direction of the cue ball, was in the positive \( y \) direction.

Momentum in the \( x \) direction

\[
m_c v_{cx} + m_b v_{bx} = m_c v'_{cx} + m_b v'_{bx}
\]

\[
v'_{bx} = \left( \frac{m_c(v_{cx} - v'_{cx})}{m_b} \right) + \frac{m_b v'_{bx}}{m_c}
\]

\[
v'_{bx} = \frac{0.165 \text{ kg}(0 - 3.7 \text{ m/s} \sin 40^\circ)}{0.155 \text{ kg}} + 0
\]

\[
v'_{bx} = -2.532 \text{ m/s}
\]

Momentum in the \( y \) direction

\[
m_c v_{cy} + m_b v_{by} = m_c v'_{cy} + m_b v'_{by}
\]

\[
v'_{by} = \left( \frac{m_c(v_{cy} - v'_{cy})}{m_b} \right) + \frac{m_b v'_{by}}{m_c}
\]

\[
v'_{by} = \frac{0.165 \text{ kg}(6.2 \text{ m/s} - 3.7 \text{ m/s} \cos 40^\circ)}{0.155 \text{ kg}} + 0
\]

\[
v'_{by} = 3.583 \text{ m/s}
\]

Pythagorean theorem and tangent function

\[
|v'_{b}|^2 = v'_{bx}^2 + v'_{by}^2
\]

\[
\tan \theta = \frac{v'_{by}}{v'_{bx}}
\]

Negative \( x \) and positive \( y \) place the angle in the second quadrant.

\[
\tan \theta = \frac{3.583 \text{ m/s}}{2.532 \text{ m/s}}
\]

\[
\theta = \tan^{-1}(1.4151)
\]

\[
\theta = 54.75^\circ
\]

\[
\theta \equiv 55^\circ \text{ clockwise from negative } x\text{-axis}
\]

30. (a) The final velocity of the blue ball is 0.29 m/s\([W21^\circ N]\).

Momentum in \( x \) direction

\[
m_c v_{rx} + m_b v_{rx} = m_c v'_{rx} + m_b v'_{rx}
\]

\[
v'_{rx} = \left( \frac{m_c(v_{rx} - v'_{rx})}{m_b} \right) + \frac{m_b v'_{rx}}{m_c}
\]

\[
v'_{rx} = \frac{0.750 \text{ kg}(0.30 \text{ m/s}[E] - 0.15 \text{ m/s} \cos 30^\circ[E])}{0.550 \text{ kg}} + 0.5 \text{ m/s}[W]
\]

\[
v'_{rx} = 0.232 \text{ m/s}[E] + 0.5 \text{ m/s}[W]
\]

\[
v'_{rx} = 0.268 \text{ m/s}[W]
\]

Momentum in \( y \) direction

\[
m_c v_{ry} + m_b v_{ry} = m_c v'_{ry} + m_b v'_{ry}
\]

\[
v'_{ry} = \left( \frac{m_c(v_{ry} - v'_{ry})}{m_b} \right) + \frac{m_b v'_{ry}}{m_c}
\]

\[
v'_{ry} = \frac{0.750 \text{ kg}(0 \text{ m/s}[E]) - 0.15 \text{ m/s} \sin 30^\circ[S]}{0.550 \text{ kg}} + 0
\]

\[
v'_{ry} = -0.1023 \text{ m/s}[S]
\]

\[
v'_{ry} = 0.1023 \text{ m/s}[N]
\]
Pythagorean theorem and tangent function

\[ |\vec{v}_{b}'|^2 = v_{bx}'^2 + v_{by}'^2 \]

\[ |\vec{v}_b|^2 = (\text{-0.268 m/s})^2 + (0.1023 \text{ m/s})^2 \]

\[ |\vec{v}_b'| = 0.29 \text{ m/s} \]

\[ \tan \theta = \frac{v_{by}'}{v_{bx}'} \]

A positive \( y \)- and negative \( x \)-component place the resultant vector in the second quadrant.

\[ \tan \theta = \frac{0.1023 \text{ m/s}}{0.268 \text{ m/s}} \]

\[ \theta = \tan^{-1}(0.38172) \]

\[ \theta \approx 21^\circ \]

(b) Kinetic energy lost in the collision of the red ball and the blue ball: 69%

Calculate the kinetic energy before the collision.

\[ E_{kr} = \frac{1}{2} m_r v_r^2 \]

\[ E_{kr} = \frac{1}{2} (0.750 \text{ kg})(0.30 \text{ m/s})^2 \]

\[ E_{kr} = 0.03375 \text{ J} \]

\[ E_{kb} = \frac{1}{2} m_b v_b^2 \]

\[ E_{kb} = \frac{1}{2} (0.550 \text{ kg})(0.50 \text{ m/s})^2 \]

\[ E_{kb} = 0.06875 \text{ J} \]

\[ E_{kr} + E_{kb} = 0.1025 \text{ J} \]

Calculate the kinetic energy after the collision.

\[ E_{kr}' = \frac{1}{2} m_r v_r'^2 \]

\[ E_{kr}' = \frac{1}{2} (0.750 \text{ kg})(0.15 \text{ m/s})^2 \]

\[ E_{kr}' = 0.00844 \text{ J} \]

\[ E_{kb}' = \frac{1}{2} m_b v_b'^2 \]

\[ E_{kb}' = \frac{1}{2} (0.550 \text{ kg})(0.29 \text{ m/s})^2 \]

\[ E_{kb}' = 0.02313 \text{ J} \]

\[ E_{kr}' + E_{kb}' = 0.03157 \text{ J} \]

Calculate percent total kinetic energy lost.

\[ \left( \frac{0.1025 \text{ J} - 0.03157 \text{ J}}{0.1025 \text{ J}} \right) \times 100\% \]

\[ \approx 69\% \]